2. *Misalignment-Compensating Couplings.* Such couplings are required for connecting two members of a power-transmission or motion-transmission system that are not perfectly aligned. "Misalignment" means that components that are coaxial by design are not actually coaxial, due either to assembly errors or to deformations of subunits and/or their foundations, Figure 2. The latter factor is of substantial importance for transmission systems on nonrigid foundations.

If the misaligned shafts are rigidly connected, this leads to elastic deformations of the shafts, and thus to dynamic loads on bearings, vibrations, increased friction losses in power transmission systems, and unwanted friction forces in motion transmission, especially in control systems.

Misalignment-compensating couplings are used to reduce the effects of imperfect alignment by allowing nonrestricted or partially restricted motion between the connected shaft ends. Similar coupling designs are sometimes used to change bending natural frequencies/modes of long shafts.

When only misalignment compensation is required, rigidity in torsional direction is usually a positive factor, otherwise the dynamic characteristics of the transmission system might be distorted. To achieve high torsional rigidity together with high mobility/compliance in misalignment directions (radial or parallel offset, axial, angular), torsional and misalignment-compensating displacements in the coupling *have to be separated* by using an intermediate compensating member. Frequently, torsionally rigid "misalignment-compensating" couplings, such as gear couplings, are referred to in the trade literature as "flexible" couplings.

3. *Torsionally Flexible Couplings.* Such couplings are used to change the dynamic characteristics of a transmission system, such as natural frequency, damping and character/degree of nonlinearity. The change is desirable or necessary when severe torsional vibrations are likely to develop in the transmission system, leading to dynamic overloads in power-transmission systems.

Torsionally flexible couplings usually demonstrate high torsional compliance to enhance their influence on transmission dynamics.

4. *Combination Purpose Couplings* are required to possess both compensating ability and torsional flexibility. The majority of the commercially available connecting couplings belong to this group.

3.1 Rigid Couplings

Typical rigid couplings are shown in Figure 3. Usually, such a coupling comprises a sleeve fitting snugly on the connected shafts and positively connected with each shaft by pins, Figure 3a, or by keys, Figure 3b. Sometimes two sleeves are used, each positively attached to one of the shafts and connected between themselves using flanges, Figure 3c. Yet another popular embodiment is the design in Figure 3d wherein the sleeve is split longitudinally and "cradles" the connected shafts.

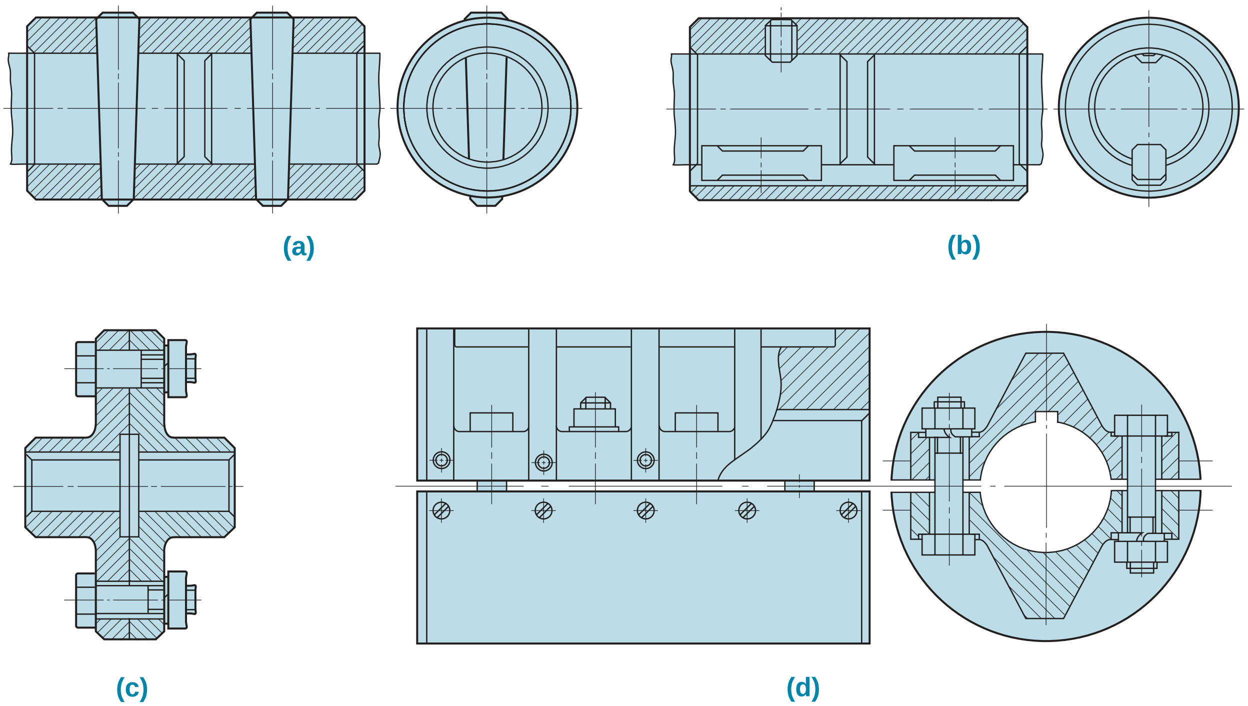

Figure 3 Examples of Rigid Couplings

3.2 Misalignment-Compensating Couplings

Misalignment-compensating couplings have to reduce forces caused by an imperfect alignment of connected rotating members (shafts). Since components which are designed to transmit higher payloads can usually tolerate higher misalignment-caused loads, a ratio between the load generated in the basic misalignment direction (radial or angular) to the payload (rated torque or tangential force) seems to be a natural design criterion for purely misalignment-compensating couplings.

All known designs of misalignment-compensating (torsionally rigid) couplings are characterized by the presence of an intermediate (floating) member located between the hubs attached to the shafts being connected. The floating member has mobility relative to both hubs. The compensating member can be solid or composed of several links. There are two basic design subclasses:

(2a) Couplings in which the displacements between the hubs and the compensating member have a frictional character (examples: Oldham coupling, Figure 4; universal Cardan Joint, Figure 5; gear coupling, Figure 6.)

(2b) Couplings in which the displacements are due to elastic deformations in special elastic connectors (e.g., "K" Type Flexible Coupling, Figure 7).

3.2.1 Selection Criterion for Frictional Misalignment-Compensating Couplings

For Subclass *(2a)* couplings designed for compensating the offset misalignment, the radial force F_{com} acting from one hub to another and caused by misalignment, is a friction force equal to the product of friction coefficient f and tangential force F_t at an effective radius R_{ef}, $F_t = T/R_{ef}$, where T is transmitted torque,

$$F_{com} = f F_t = \frac{f T}{R_{ef}} \tag{1}$$

Since motions between the hubs and the compensating member are of a "stick-slip" character, with very short displacements alternating with stoppages and reversals, f might be assumed to be the static friction coefficient.

When the rated torque T_r is transmitted, then the selection criterion is

$$\frac{F_{com}}{T_r} = \frac{f}{R_{ef}} \tag{2}$$

or the ratio representing the selection criterion does not depend on the amount of misalignment; lower friction and/or larger effective radius would lead to lower forces on bearings of the connected shafts.

Similar conclusion stands for couplings compensating angular misalignments (Cardan joints or universal, or simply, U-joints). While U-joints with rolling friction (usually, needle) bearings have low friction coefficient, f for U-joints with sliding friction can be significant if the lubrication system is not properly designed and maintained.

3.2.1a Oldham Couplings

Oldham couplings consist of three members. A floating member is trapped by 90° displaced grooves between the two outer members which connect to the drive shafts, as shown in Figure 4.

Oldham couplings can accommodate lateral shaft misalignments up to 10% of nominal shaft diameters and up to 3° angular misalignments.

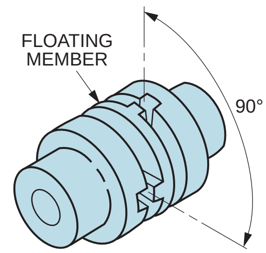

Figure 4 Oldham Coupling

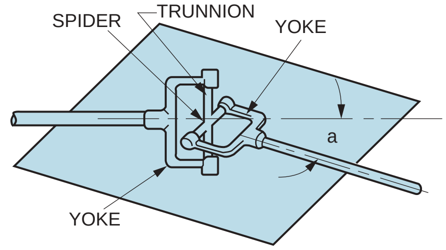

Figure 5 Universal Cardan Joint

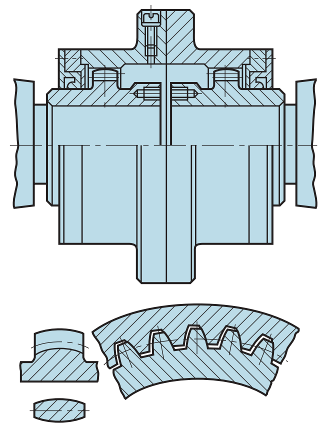

Figure 6 Gear Coupling

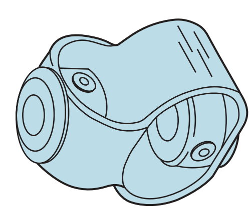

Figure 7 K-Type Elastomeric Coupling/Joint

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Lubrication is a problem but can, in most applications, be overcome by choosing a coupling that uses a wear-resistant plastic in place of steel or bronze floating members.

Some advantages of Oldham couplings:

- High torsional stiffness;
- No velocity variation as with universal joints;
- Substantial lateral misalignments possible;
- High torque capacity for a given size;
- Ease of disassembly.

Shortcomings of Oldham couplings:

- Limited angular misalignment of shafts;
- Need for lubrication due to relative sliding motion with stoppages, unless wear-resistant plastic is employed;
- Nylon coupling has reduced torque capacity;
- Significant backlash due to initial clearances for thermal expansion and inevitable wear;
- Are not suitable for small misalignments;
- Suitable only for relatively slow-speed transmissions;
- Possible loss of loose members during disassembly.

Oldham couplings with rubber-metal laminated bearings [1] have all the advantages of the generic Oldham couplings without their shortcomings.

3.2.1b Universal or U-joints [2]

3.2.1b.1 General

A universal joint, Figure 5, is a positive, mechanical connection between rotating shafts, which are not parallel, but intersecting. It is used to transmit motion, power, or both. It is also called the *Cardan joint* or *Hooke joint*. It consists of two *yokes*, one on each shaft, connected by a cross-shaped intermediate member called the *spider* having four *trunnions* providing for rotatable connections with the yokes. The angle between the two shafts is called the *operating angle*. It is generally, but not always, constant during operation. Good design practice calls for low operating angles, often less than 25°, depending on the application. Independent of this guideline, mechanical interference in the U-joint designs often limits the operating angle to a maximum (usually about 37.5°), depending on its proportions.

Typical applications of U-joints include aircraft, appliances, control mechanisms, electronics, instrumentation, medical and optical devices, ordnance, radio, sewing machines, textile machinery and tool drives.

U-joints are available with steel or plastic major components. Steel U-joints have maximum load-carrying capacity for a given size. U-joints with plastic body members are used in light industrial applications in which their self-lubricating feature, light weight, negligible backlash, corrosion resistance and capability for high-speed operation are significant advantages.

Recently developed U-joint designs with *rubber-metal laminated bearings* [1, 3] have even higher torque capacity and/or smaller sizes allowing for higher-speed operation, and can be preloaded without increasing friction losses, thus completely eliminating backlash. These designs do not require lubrication and sealing against contamination.

Constant velocity or *ball-jointed* universals are also available. These are used for high-speed operation and for carrying large torques. They are available in both miniature and standard sizes.

Motion transmitted through a U-joint becomes nonuniform. The angular velocity ratio between input and output shafts varies cyclically (two cycles per one revolution of the input shaft). This fluctuation, creating angular accelerations and increasing with the increasing angular misalignment, can be as much as ±15% at 30° misalignment. Effects of such fluctuations on static torque, inertia torque, and overall system performance should be kept in mind during the transmission design.

This nonuniformity can be eliminated (canceled) by using two connected in series and appropriately phased U-joints, Figure 8. While the output velocity becomes uniform, angular velocity fluctuation of the intermediate shaft cannot be avoided.

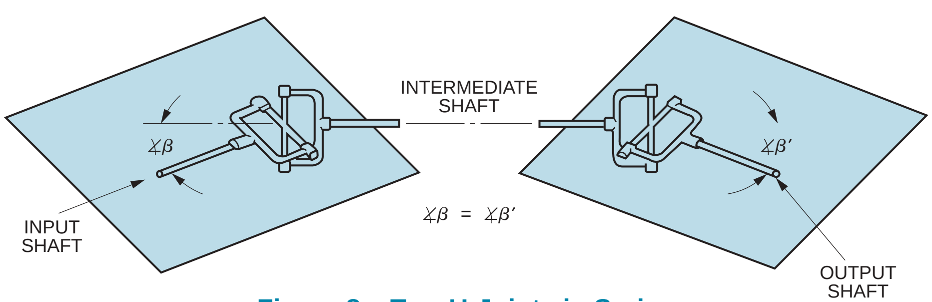

Figure 8 Two U-Joints in Series

Two U-joints in series can be used for coupling two laterally displaced (misaligned) shafts, while the single joint can only connect the angularly-misaligned shafts.

Advantages of a single U-joint:

- Low side thrust on bearings;
- Large angular misalignments are possible;
- High torsional stiffness;
- High torque capacity.

Shortcomings of a single U-joint:

- Velocity and acceleration fluctuations, especially for large misalignments;
- Lubrication is required to reduce friction and wear;
- Protection from contamination (sealing) is required;
- Shafts must be precisely located in one plane;
- Backlash is difficult to control;
- Static friction is increasing at very low misalignment (freezing), thus sometimes requiring an artificial misalignment in the assembly.

3.2.1b.2 Kinematics

Due to the velocity fluctuations, the angular displacements of the output shaft do not precisely follow those of the input shaft, but lead or lag, also with two cycles per revolution. The angular velocity variation is shown in Figure 9 for several operating (misalignment) angles β. The peak values of the displacement lead/lag, of input/output angular velocity ratio, and of angular acceleration ratio for different α are given in Table 1 [2]. As a qualitative guideline, for small β, up to ~10°, the deviations (errors) for maximum lead/lag angular displacements, for maximum deviations of angular velocity ratios from unity, and for maximum angular acceleration ratios are nearly proportional to the square of β.

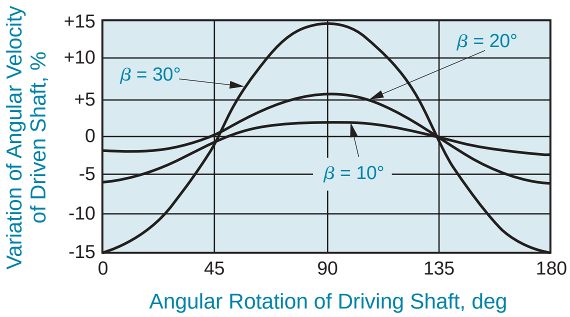

Figure 9 Angular Velocity Variations in U-Joint

The static torque transmitted by the output shaft is equal to the product of the input torque and the angular velocity ratio.

The angular acceleration generates inertia torque and vibrations. The total transmitted torque is a sum of inertia torque (the product of the angular acceleration and the mass moment of inertia of the output shaft and masses associated with it) and the nominal output torque.

The inertia torque often determines the ultimate speed limit of the joint. The recommended speed limits vary depending on β, on transmitted power, and on the nature of the transmission system. Recommended peak angular accelerations of the driven shaft vary from 300 rad/sec^2 to over 2000 rad/sec^2 in power drives. In light instrument drives, the allowable angular accelerations may be higher. For an accurate determination of the allowable speed, a stress analysis is necessary.

Example 1: Determining the Maximum Inertia Torque

A U-joint operates at 250 rpm with an operating angle β = 10°. Find the maximum angular displacement lead (or lag), maximum and minimum angular velocity of output shaft and maximum angular acceleration of output shaft.

If the system drives an inertial load so that the total inertial load seen by the output shaft (its own inertia and inertia of associated massive rotating bodies) can be represented by a steel circular disc attached to the output shaft (radius r = 3 in., thickness t = 1/4 in.), find the maximum inertia torque of the drive.

From Table 1 at β = 10°, the maximum displacement lead/lag = 0.439° = 26.3'. The maximum and minimum angular velocity ratios are given as 1.0154 and 0.9848, respectively. Hence, the corresponding output shaft speeds are:

$$\Omega_{max} = (250)(1.0154) = 254 \text{ rpm};$$

$$\Omega_{min}\ (250)(0.9848) = 246 \text{ rpm};$$

According to Table 1, the maximum angular acceleration ratio is

$$\alpha_{max}/\omega^2 = 0.0306 \text{ for } \beta = 10°.$$

$$\omega = [(250)\ (2\pi)]\ /\ (60) \text{ rad/sec} = 26.18 \text{ rad/sec}.$$

Hence, $\alpha_{max} = (0.0306)(26.18)^2 = 21.0$ rad/sec^2. The weight, W, of the disc is given by W = π r^2 t γ, where γ denotes the density of steel and is equal to 0.283 lb/in^3.

TABLE 1 THE EFFECT OF SHAFT ANGLE (β) ON SINGLE UNIVERSAL JOINT PERFORMANCE FOR CONSTANT INPUT SPEED*

Operating Angle Between Shafts (β) Deg.	Maximum Load or Lag of Output Shaft Displacement (ε), Deg. Relative to Input Shaft Displacement	Maximum Angular Velocity Ratio (Ω_{max})	Minimum Angular Velocity Ratio (Ω_{min})	Maximum Angular Acceleration Ratio = $\frac{\alpha_{max}}{\omega^r}$, where α_{max} = Maximum Angular Acceleration of Output Shaft; ω = Angular Velocity of Input Shaft, rad/sec.
0	0.000	1.0000	1.0000	0.0000
1	0.004	1.0002	0.9998	0.0003
2	0.017	1.0006	0.9994	0.0012
3	0.039	1.0014	0.9986	0.0027
4	0.070	1.0024	0.9976	0.0049
5	0.109	1.0038	0.9962	0.0076
6	0.157	1.0055	0.9945	0.0110
7	0.214	1.0075	0.9925	0.0150
8	0.280	1.0098	0.9903	0.0196
9	0.355	1.0125	0.9877	0.0248
10	0.439	1.0154	0.9848	0.0306
11	0.531	1.0187	0.9816	0.0371
12	0.633	1.0223	0.9781	0.0442
13	0.744	1.0263	0.9744	0.0520
14	0.864	1.0306	0.9703	0.0604
15	0.993	1.0353	0.9859	0.0694
16	1.132	1.0403	0.9613	0.0792
17	1.280	1.0457	0.9563	0.0896
18	1.437	1.0515	0.9511	0.1007
19	1.605	1.0576	0.9455	0.1125
20	1.782	1.0642	0.9397	0.1250
21	1.969	1.0711	0.9336	0.1382
22	2.165	1.0785	0.9272	0.1522
23	3.372	1.0864	0.9205	0.1670
24	2.590	1.0946	0.9135	0.1826
25	2.817	1.1034	0.9063	0.1990
26	3.055	1.1126	0.8988	0.2162
27	3.304	1.1223	0.8910	0.2344
28	3.564	1.1326	0.8829	0.2535
29	3.835	1.1434	0.8746	0.2735
30	4.117	1.1547	0.8660	0.2946
31	4.411	1.1666	0.8572	0.3167
32	4.716	1.1792	0.8480	0.3400
33	5.034	1.1924	0.8387	0.3644
34	5.363	1.2062	0.8290	0.3902
35	5.705	1.2208	0.8192	0.4172
36	6.060	1.2361	0.8090	0.4457
37	6.428	1.2521	0.7986	0.4758
38	6.809	1.2690	0.7880	0.5074
39	7.204	1.2868	0.7771	0.5409
40	7.613	1.3054	0.7660	0.5762

*Reproduced with the permission of Design News from "The Analytical Design of Universal Joints" by S.J. Baranyi, Design News, Sept. 1, 1969

$W = \pi (3)^2 (0.25) (0.283) = 2$ lb.

Inertia torque = $I\alpha_{max}$, where I = polar mass moment of inertia of disc (lb. in. sec^2),

$I = Wr^2 / 2g$,

where g = gravitational constant = 386 in/sec^2.

Hence, $I = [(2) (3)^2] / [(2)(386)] = 0.0233$ lb. in. sec^2.

Inertia torque = (21.0) (0.0233) = 0.489 lb. in. This inertia torque is a momentary maximum. The inertia torque fluctuates cyclically at two cycles per shaft revolution, oscillating between plus and minus 0.489 lb. in.

When system vibrations and resonances are important, it may be required to determine the harmonic content (Fourier series development) of the output shaft displacement as a function of the displacement of the input shaft. The amplitude of the m^{th} harmonic ($m > 1$) vanishes for odd values of m, while for even values of m it is equal to $(2/m) (\tan 1/2\beta)^m$, where β denotes the operating angle.

3.2.1b.3 Joint Selection (Torque Rating)

The torque capacity of the universal joint is a function of speed, operating angle and service conditions. Table 2 shows use factors based on speed and operating angle for two service conditions: *intermittent* operation (say, operation for less than 15 minutes, usually governed by necessity for heat dissipation) and *continuous* operation.

TABLE 2 USE FACTORS FOR THE TORQUE RATING OF UNIVERSAL JOINTS

Speed rpm	Intermittant Running Conditions								
	Angle of Operation - Degrees								
	0	3	5	7	10	15	20	25	30
1800	9	20	34	45	—	—	—	—	—
1500	8	16	28	39	—	—	—	—	—
1200	7	13	22	32	40	—	—	—	—
900	6	11	16	23	34	—	—	—	—
600	5	8	11	15	22	34	40	—	—
300	4	5	7	8	11	16	22	28	34
100	3	4	4	5	6	8	9	11	12

Speed rpm	Continuous Running Conditions								
	Angle of Operation - Degrees								
	0	3	5	7	10	15	20	25	30
1800	18	40	68	90	—	—	—	—	—
1500	16	32	55	78	—	—	—	—	—
1200	14	26	44	64	80	—	—	—	—
900	12	21	32	46	68	—	—	—	—
600	10	15	22	30	44	68	80	—	—
300	8	10	14	16	22	32	44	55	68
100	6	7	8	10	12	15	18	22	24

The torque capacity of a single Cardan joint of standard steel construction is determined as follows:

i. From the required speed (rpm), operating angle in degrees, and service condition (intermittent or continuous), find the corresponding use factor from Table 2.

ii. Multiply the required torque, which is to be transmitted by the input shaft, by the use factor. If the application involves a significant amount of shock loading, multiply by an additional dynamic factor of 2. The result must be less than the static breaking torque of the joint.

iii. Refer to the torque capacity column in the product catalog and select a suitable joint having a torque capacity not less than the figure computed in (ii) above.

If a significant amount of power is to be transmitted and/or the speed is high, it is desirable to keep the shaft operating angle below 15°. For manual operation, operating angles up to 30° may be permissible.

Example 2: Universal Joint Selection for Continuous Operation

A single universal joint is to transmit a continuously acting torque of 15 lb. in., while operating at an angle of 15° and at a speed of 600 rpm. Select a suitable joint.

From Table 2 for continuous operation, the use factor is given as 68. Note that there are blank spaces in the Table. If the combination of operating angle and speed results in a blank entry in the Table, this combination should be avoided. The required torque is (68) (15) = 1020 lb. in. There is no shock load and the dynamic factor of 2 does not apply in this case.

From the SDP/SI catalog, it is seen that there are two joints meeting this specification: A 5Q 8-D500 and A 5Q 8-D516, both with a torque capacity of 1176 lb. in. The first has a solid-shaft construction and the second a bored construction. The choice depends on the application.

Example 3: Universal Joint Selection for Intermittent Operation with Shock Loading

A single universal joint is to transmit 1/8 horsepower at 300 rpm at an operating angle of 15°. Select a suitable joint for intermittent operation with shock loading.

Here we make use of the equation:

Torque = Horsepower x 63,025/300 lb. in.

Hence, operating torque = (0.125)(63,025)/300 = 26.3 lb. in. From Table 2, for intermittent loads (300 rpm, 15°), the use factor is 16. Due to shock loading, there should be an additional dynamic factor of 2. Therefore, the rated torque = (26.3) (16) (2) = 842 lb. in. Thus, the same joints found in the previous example are usable in this case.

Example 4: Determining the Maximum Speed of an Input Shaft

A universal joint is rated at 250 lb. in., and operates at an angle of 12°, driving a rotating mass, which can be represented (together with the inertia of the driven shaft) by a steel, circular disc, radius r = 6", thickness t = 1/2", attached to the driven shaft. How fast can the input shaft turn if the inertia torque is not to exceed 50% of rated torque?

From Table 1, for β = 12°, we have α_{max}/ω^2 = 0.0442. The weight, W, of the disc is W = $\pi r^2 t \gamma$, where γ denotes the density of steel which is 0.283 lb. in^3.

Thus W = π (6)^2(0.5) (0.283) = 16 lb. The polar mass moment of inertia, I, of the disc is given by

$$I = Wr^2 / 2g = (16)(6)^2 / (2)(386) = 0.746 \text{ lb. in. sec}^2.$$

The inertia torque = $I\alpha_{max}$ = 50% of 250 lb. in. = 125 lb. in. Since $I\alpha_{max} = (\alpha_{max} / \omega^2) \bullet (\omega^2 I) = (0.0442)(0.746)\,\omega^2 = 125$, $\omega^2 = 125 / 0.03297 = 3790.96$ or ω = 61.6 rad/sec = (61.6)(60) / 2π = 588 rpm.

Hence, if the inertia torque is not to exceed its limit, the maximum speed of the input shaft is 588 rpm. For joints made with thermoplastic material, consult the SDP/SI catalog, which contains design charts for the torque rating of such joints.

3.2.1b.4 Secondary Couples

In designing support bearings for the shafts of a U-joint and in determining vibrational characteristics of the driven system, it is useful to keep in mind the so-called *secondary couples* or *rocking torques*, which occur in universal joints. These are rocking couples in the planes of the yokes, which tend to bend the two shafts and rock them about their bearings. The bearings are thus cyclically loaded at the rate of two cycles per shaft revolution. The maximum values of the rocking torques are as follows:

Maximum rocking torque on input shaft = $T_{in}\tan\beta$;
Maximum rocking torque on output shaft = $T_{in}\sin\beta$,

where T_{in} denotes the torque transmitted by the input shaft and β the operating angle. These couples are always 180° out of phase. The bearing force induced by these couples is equal to magnitude of the rocking couple divided by the distance between shaft bearings.

For example, if the input torque, T_{in} is 1000 lb. in. and the operating angle is 20°, while the distance between support bearings on each shaft is 6 in., the maximum secondary couple acting on the input shaft is (1000) (tan 20°) = 364 lb. in. and on the output shaft it is (1000) (sin 20°) = 342 lb. in. The radial bearing load on each bearing of the input shaft is 364/6 = 60.7 lb. and it is 342/6 = 57 lb. for each bearing of the output shaft. The bearings should be selected accordingly.

It has been observed also that due to the double frequency of these torques, the critical speeds associated with universal drives may be reduced by up to 50% of the value calculated by the standard formulas for the critical speeds of rotating shafts. The exact percentage is a complex function of system design and operating conditions.

3.2.1b.5 Joints in Series

As mentioned in paragraph 3.2.1b.1, universal joints can be used in series in order to eliminate velocity fluctuations, to connect offset (nonintersecting) shafts, or both. Figure 8 shows a schematic of such an arrangement.

In order to obtain a constant angular-velocity ratio (1:1) between input and output shafts, a proper phasing of the joints is required. This phasing can be described as follows: two Cardan joints in series will transmit a constant angular velocity ratio (1:1) between two intersecting or nonintersecting shafts (see Figure 8), provided that the angle between the connected shafts and the intermediate shaft are equal ($\beta = \beta'$), and that when yoke 1 lies in the plane of the input and intermediate shafts, yoke 2 lies in the plane of the intermediate shaft and the output shaft.

If shafts 1 and 3 intersect, yokes 1 and 2 are coplanar.

When the above phasing has been realized, torsional and inertial excitation is reduced to minimum. However, inertia excitation will inevitably remain in the intermediate shaft, because this shaft has the angular acceleration of the output shaft of a single U-joint (the first of the two joints in series). It is for this reason that guidelines exist limiting the maximum angular accelerations of the intermediate shaft. Depending on the application, values between 300 rad/sec^2 and values in excess of 1000 rad/sec^2 have been advocated. In light industrial drives, the allowable speed may be higher. For an accurate determination of allowable speed, a stress analysis is necessary.

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Example 5: Determining the Maximum Speed of an Input Shaft in a Series

In a drive consisting ot two universal joints in series, phased so as to produce a constant (1:1) angular velocity ratio between input and output shafts, the angle between the intermediate shaft and input (and output) shaft is 20°. If the maximum angular acceleration of the intermediate shaft is not to exceed 1000 rad/sec^2, what is the upper limit of the speed of the input shaft?

From Table 1, with β = 20°, we find $\alpha_{max}/\omega^2 = 0.1250$.

Since α_{max} = 1000 rad/sec^2,

$\omega^2 = (\alpha_{max}) / (0.1250) = (1000) / 0.1250 = 8000$ rad/sec^2.

Hence, $\omega = \sqrt{8000}$ = 89.4 rad/sec = (89.4)(60) / 2π = 854 rpm.

Hence, the speed of the input shaft should not exceed 854 rpm. When the joint angle is less than or equal to 10°, Figure 10 can be used to compute the maximum speed or the maximum angular acceleration for a given input speed.

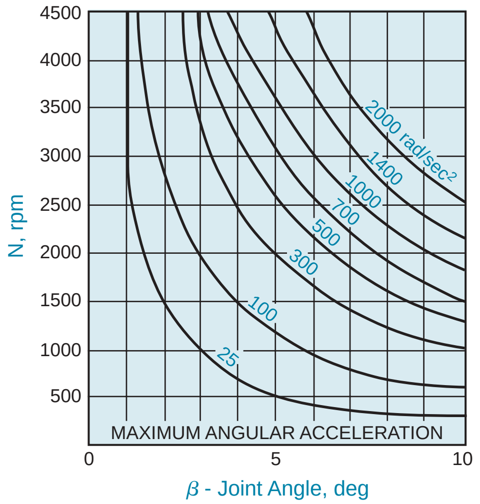


Figure 10 Maximum Angular Acceleration (rad/sec^2) of Output Shaft of Single U-Joint vs. Input Speed (rpm) and Operating Angle (degrees)

Example 6

Same as problem 5, except operating angle is 10°. Here we can use Figure 10. The intersection of $\beta = 10°$ and the 1000 rad/sec^2 curve yields N $\approx$ 1800 rpm. Hence, the speed of the input shaft should not exceed 1800 rpm. A more exact calculation, as in Example 5, yields N = 1726 rpm. For practical purposes, however, the value obtained from Figure 10 is entirely satisfactory.

3.2.2 Selection Criterion for Misalignment-Compensating Couplings with Elastic Connectors

For this class of couplings, assuming linearity of the elastic connectors,

$$F_{com} = k_{com}e, \tag{3}$$

where e is misalignment value, k_{com} = combined stiffness of the elastic connectors in the direction of compensation. In this case,

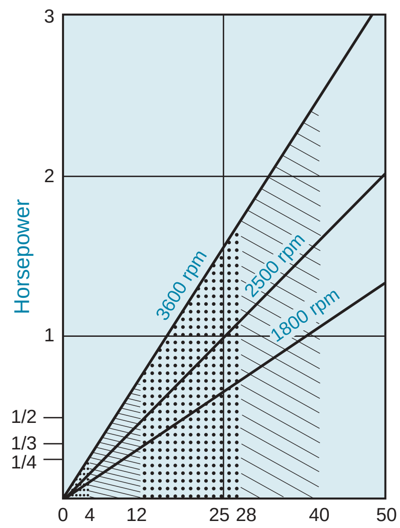
$$\frac{F_{com}}{T_r} = \frac{k_{com}}{T_r}e. \tag{4}$$

Unlike couplings from Subclass (*2a*), Subclass (*2b*) (see p. T2-5) couplings develop the same radial force for a given misalignment regardless of transmitted torque, thus they are more effective for larger T_r. Of course, lower stiffness of the elastic connectors would lead to lower radial forces.

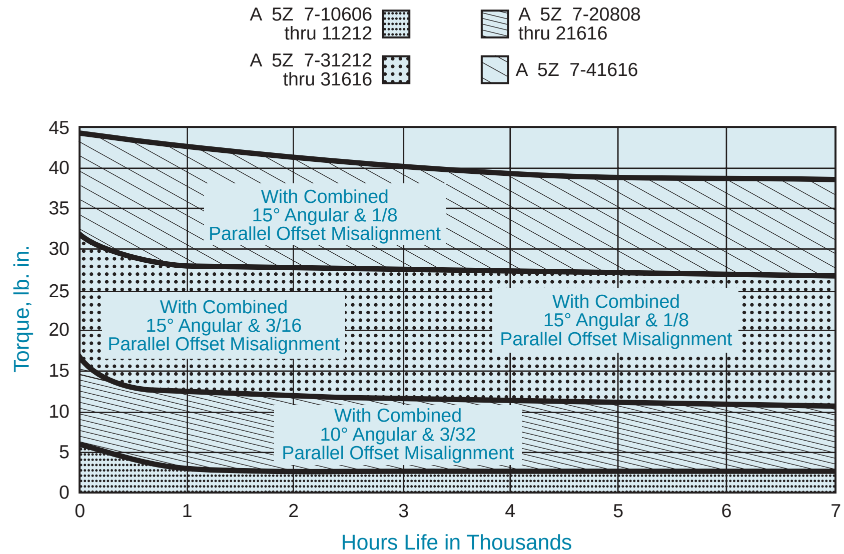
3.2.2.1 Designs of Elastic Misalignment-Compensating Couplings

Designs of Oldham couplings and U-joints with elastic connectors using high-performance thin-layered rubber-metal laminates are described in [1, 3].

K-Type flexible coupling, Figure 7, is kinematically similar to both Oldham coupling and to U-joint. By substituting an elastomeric member in place of the conventional spider and yoke of U-joint or the floating member of Oldham coupling, in construction such as in the design shown in Figure 7, backlash is eliminated. Lubrication is no longer a consideration because there are no moving parts and a fairly large amount of lateral misalignment can be accommodated. The illustrated coupling is available in the product section of this catalog. Please refer to Figure 11 for specific design data for four sizes of this type coupling. Figure 11b indicates that this coupling has high durability even with a combination of large lateral (offset) and large angular misalignments.



(a) Rated Horsepower/ Torque for Various rpm



(b) Service Life as a Function of Angular and Offset Misalignments for K-Type Couplings

Figure 11

3.3 Torsionally Flexible Couplings and Combination Purpose Couplings [3]

These two classes of couplings are usually represented by the same designs. However, in some cases only torsional properties are required, in other cases both torsional and compensation properties are important and, most frequently, these coupling designs are used as the cheapest available and users cannot determine what is important for their applications. Accordingly, it is of interest to look at what design parameters are important for various applications.

3.3.1 Torsionally Flexible Couplings

Torsionally flexible couplings are used in transmission systems when there is a danger of developing resonance conditions and/or transient dynamic overloads. Their influence on transmission dynamics can be due to one or more of the following factors:

Reduction of Torsional Stiffness and, Consequently, Shift of Natural Frequencies.

If resonance condition occurs before installation (or change) of the coupling, then shifting of the natural frequency can eliminate resonance; thus dynamic loads and torsional vibrations will be substantially reduced.

Increasing Effective Damping Capacity of a Transmission by Using Coupling Material with High Internal Damping or Special Dampers.

When the damping of a system is increased without changing its torsional stiffness, the amplitudes of torsional vibrations are reduced at resonance and in the near-resonance zone. Increased damping is especially advisable when there is a wide frequency spectrum of disturbances acting on a drive; more specifically, for the drives of universal machines.

Introducing Nonlinearity into the Transmission System.

If the coupling has a nonlinear "torque-angular deformation" characteristic and its stiffness is much lower then stiffness of the transmission into which it is installed, then the whole transmission acquires a nonlinear torque-angular deflection characteristic. A nonlinear dynamic system becomes automatically detuned away from resonance at a fixed-frequency excitation, the more so the greater the relative change of the overall stiffness of the system on the torsional deflection equal to the vibration amplitude.

Introducing Additional Rotational Inertia in the Transmission System.

This is a secondary effect since couplings are not conventionally used as flywheels. However, when a large coupling is used, this effect has to be considered.

Realizing the above listed effects of a properly selected torsionally-flexible coupling requires a thorough dynamic analysis of the transmission system.

3.3.2 Combination Purpose Couplings

Combination purpose couplings do not have a special compensating (floating) member. As a result, compensation of misalignment is accomplished, at least partially, by the same mode(s) of deformation of the flexible element which are called forth by the transmitted payload.

The ratio of radial (compensating) stiffness k_{com} and torsional stiffness k_{tor} of a combination purpose flexible coupling can be represented as [1,3]

$$\frac{k_{com}}{k_{tor}} = \frac{A}{R_{ex}^2} \quad , \tag{5}$$

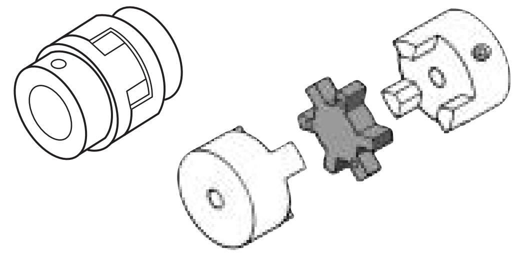
where R_{ex} is external radius of the coupling. The "Coupling Design Index" A (Figure 13f) allows one to select a coupling design better suited to a specific application. If the main purpose is to reduce misalignment-caused loading of the connected shafts and their bearings, for a given value of torsional stiffness, then the lowest value of A is the best, together with large external radius. If the main purpose is to modify the dynamic characteristics of the transmission, then minimization of k_{tor} is important.

Some combination purpose couplings are shown in Figure 12. The "modified spider" coupling (Figure 12b) is different from the conventional spider (jaw) coupling shown schematically in Figure 12a by four features: legs of the rubber spider are tapered, instead of staight; legs are made thicker even in the smallest cross section, at the expense of reduced thickness of bosses on the hubs; lips ℓ on the edges provide additional space for bulging of the rubber when legs are compressed, thus reducing stiffness; the spider is made from a very soft rubber. All these features lead to substantially reduced torsional and radial stiffnesses while retaining small size, which is characteristic for spider couplings.

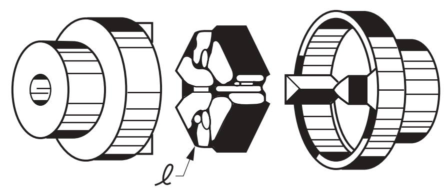
Plots in Figure 13 (a-d) give data for some widely used couplings on such basic parameters as torsional stiffness k_{tor}, radial stiffness k_{com}, external diameter D_{ex}, and flywheel moment WD^2 (W is weight of the coupling). Plots in Figure 13 (e-f) give derivative information: ratio k_{com} / k_{tor}, and design index A. All these parameters are plotted as functions of the rated torque.

Data for "toroid shell" couplings in Figure 13 are for the coupling as shown in Figure 12 (c) (there are many design modifications of toroid shell couplings). The "jaw coupling" for T = 7 Nm in Figure 13 (f) (lowest torque point on jaw coupling line) has a four-legged spider (z = 4), while all larger sizes have z = 6 or 8. This explains differences in A ($A \approx 1.9$ for z = 4, but A = 1.0 ~ 1.3 for z = 6,8). Values of A are quite consistent for a given type of coupling. The variations can be explained by differences in design proportions and rubber blends between the sizes.

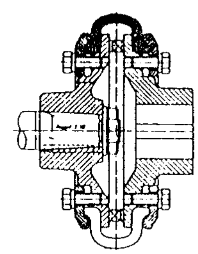
Using plots in Figure 13, one can more easily select a coupling type whose stiffnesses, inertia, and diameter are best suited for a particular application. These plots, however, do not address issues of damping and nonlinearity. Damping can be easily modified by the coupling manufacturer by a proper selection of the elastomer. As shown previously, high damping is very beneficial for transmission dynamics, and may even reduce thermal exposure of the coupling, as shown in [1,3]. More complex is the issue of nonlinear characteristics; a highly nonlinear (and very compact) coupling based on radial compression of cylindrical rubber elements is described in [1]. Couplings represented in Figure 13 are linear or only slightly nonlinear.



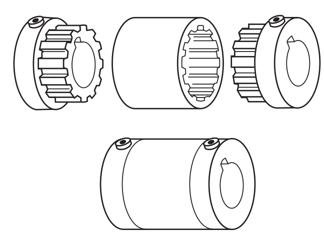
(a) Jaw (Spider) Coupling



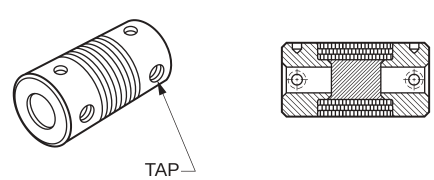
(b) Modified Spider Coupling (ℓ - lip providing bulging space for the rubber element)



(c) Toroid Shell Coupling



(d) Sleeve Coupling (Geargrip)



(e) Uniflex Coupling

Figure 12 Combination Purpose Couplings

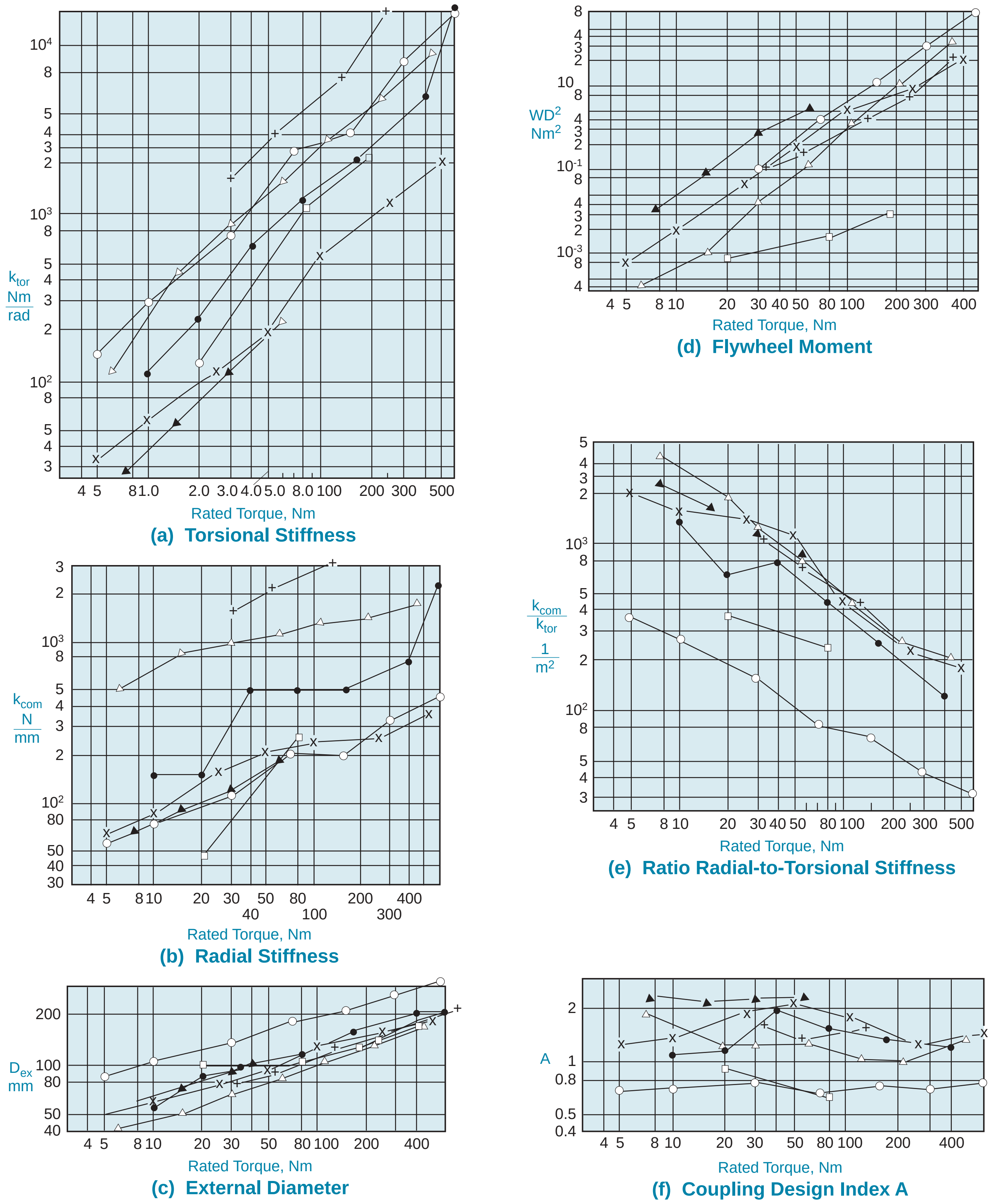


Figure 13 Basic Characteristics of Frequently Used Torsionally Flexible/Combination Purpose Couplings

Δ - Jaw Coupling with Rubber Spider	Figure 12 (a)	□ - Rubber Disk Coupling	Not Shown
▲ - Modified Spider Coupling	Figure 12 (b)	● - Uniflex Coupling	Figure 12 (e)
○ - Toroid Shell Coupling	Figure 12 (c)	+ - Finger Sleeve Coupling	Figure 12 (d)

3.3.2.1 Miscellaneous Combination Purpose Couplings

3.3.2.1a Flexible Shafts

Flexible shafts are relatively stiff in torsion but very compliant in bending and lateral misalignments. A good example of this is in their use on automotive speedometer drives.

A flexible shaft consists of:

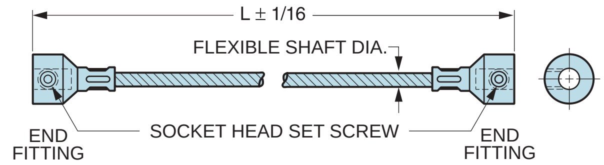


Figure 14 Flexible Shaft

a. Shaft - the rotating element comprising a center wire with several wire layers wrapped around it in alternating directions.

b. Casing - the sleeve made from metal or nonmetals to guide and protect the shaft and retain lubricants. Flexible shafts can be supplied without casing when used for hand-operated controls or intermittent-powered applications.

c. Case End Fitting - connects the casing to the housing of the driver and driven equipment.

d. Shaft End Fitting - connects the shaft to the driving and driven members. Flexible shafts as shown in the SDP/SI catalogs [4] are often substituted in place of more expensive gear trains and universal joints in applications where the load must be moved in many directions. They are extremely useful where the load is located in a remote position requiring many gear and shafting combinations.

The basic design considerations are torque capacity, speed, direction of rotation, bend radii and service conditions. Torque capacity is a function of the shaft size. Operating conditions must be considered in power drive applications such as starting torque, reversing shocks, and fluctuating loads. These conditions constitute overloads on the shaft. If they are substantially greater than the normal torque load, a larger shaft must be selected. Since, in power applications, torque is inversely proportional to speed, it is beneficial to keep the torque down, thereby reducing shaft size and cost.
Ordinarily, speeds of 1750 to 3600 rpm are recommended. However, there are applications in which shafts are operating successfully from 600 to 12,000 rpm. The general formula for determining maximum shaft speed is:

$N = (7200) / \pi d$, where N = rpm, d = shaft diameter in inches. (6)

Flexible shafting for power transmission is wound for maximum efficiency when rotating in only one direction - the direction which tends to tighten the outer layer of wires on the shaft. Direction of rotation is identified from the power source end of the shaft. Torque capacity in the opposite direction is approximately 60% of the "wind" direction. Therefore, if the power drive shaft must be operated in both directions, the reduced torque capacity will require a larger shaft than would normally be selected for operation in the wind direction.

Because flexible shafts were developed primarily as a means of transmitting power where solid shafts cannot be used, most applications involve curves. Each shaft has a recommended minimum operating radius which is determined by the shaft diameter and type. As the radius of curvature is decreased, the torque capacity also decreases and tends to shorten shaft life.

Lastly, service conditions such as temperature present no special problems to flexible shafts when operating in the -65°F to +250°F range. Plastic casing coverings are able to cover this temperature range and provide additional protection from physical abrasion as well as being oil and watertight.

3.3.2.1b Uniflex Couplings

Sometimes it is desirable if not essential that a flexible shaft coupling be as short as possible and still retain most of the features previously described. Figure 12e illustrates such a coupling, available in the SDP/SI catalog [4].

The "flexible shaft" center section consists of three separately wound square wire springs. Individual spring layers are opposingly wound to provide maximum absorption of vibration, load shock, and backlash. The hubs are brazed to the springs for maximum strength. Design data is available in Table 3 as well as in the Uniflex catalog page of the SDP/SI catalog.
The maximum torque and/or H.P. Capacity from Table 3 must be divided by the *Service Factor (S.F.)* dependent on the load character as follows:

a. Light, even load - S.F. = 1.0;
b. Irregular load without shock, rare reversals of direction - S.F. = 1.5
c. Shock loads, frequent reversals - S.F. = 2.0

TABLE 3 UNIFLEX COUPLINGS SELECTION DATA

Series Number	Max. Torque lb. in.	Horsepower Capacity* At Varying Speeds (rpm)									
		100	300	600	900	1200	1500	1800	2400	3000	3600
18	18	.03	.09	.18	.27	.36	.45	.5	.7	.9	1
25	34	.05	.15	.30	.45	.60	.75	.9	1.2	1.5	1.8
37	39	.06	.18	.36	.54	.70	.90	1	2.4	1.8	2
50	82	.13	.39	.78	1.2	1.5	2	2.3	3	3.9	4.6

*****Based on service factor of one only

Uniflex Selection Procedure:

a. Select the service factor according to the application.
b. Multiply the horsepower or torque to be transmitted by the service factor to obtain rating.
c. Select the coupling with an equivalent or slightly greater horsepower or torque than shown in Table 3.

3.3.2.1c Jaw and Spider Couplings

Jaw type couplings, Figures 12a, 12b consist of two metal hubs which are fastened to the input and output shafts (see product pages in this catalog). Trapped between the hubs is a rubber or Urethane "spider" whose legs are confined between alternating metal projections from the adjacent hubs. The spider is the wearing member and can be readily replaced without dismantling adjacent equipment. The coupling is capable of operating without lubrication and is unaffected by oil, grease, dirt or moisture. Select the proper size for your application from Table 4 and the selection instructions. The Service Factors are, essentially, the same as for the Uniflex coupling.

Jaw and Spider Type Coupling Selection Procedure:

a. Select the Service Factor according to the application.
b. Multiply the horsepower or torque to be transmitted by the service factor to obtain rating.
c. Select the coupling series from Table 4 with an equivalent or slighlty greater horsepower or torque than the calculated value in b.
d. Turn to the product section page illustrating the same coupling and make your specific selection in that number series.

TABLE 4 JAW TYPE COUPLINGS SELECTION DATA

Coupling Series Number	Rated Torque lb. in.	Service Factor	Horsepower Capacity at Varying Speeds (rpm)									
			100	300	600	900	1200	1500	1800	2400	3000	3600
035	3.5	1.0	.0056	.017	.034	.05	.067	.084	.13	.10	.17	.2
		1.5	.0037	.011	.023	.033	.045	.056	.087	.067	.113	.13
		2.0	.0028	.009	.017	.025	.033	.043	.065	.05	.025	.10
050	25.2	1.0	.04	.12	.24	.36	.48	.60	.72	.96	1.2	1.44
		1.5	.03	.08	.16	.24	.32	.40	.48	.64	.80	.96
		2.0	.02	.06	.12	.18	.24	.30	.36	.42	.60	.70
070	37.8	1.0	.06	.18	.36	.54	.72	.90	1.08	1.44	1.8	2.16
		1.5	.04	.12	.24	.36	.48	.60	.72	.96	1.2	1.44
		2.0	.03	.09	.12	.27	.36	.45	.54	.72	.90	1.08
075	75.6	1.0	.12	.36	.72	1.08	1.44	1.80	2.16	2.88	3.6	4.34
		1.5	.08	.24	.48	.72	.96	1.20	1.44	1.92	2.4	2.88
		2.0	.06	.18	.36	.54	.72	.90	1.08	1.44	1.8	2.10
090	126	1.0	.20	.60	1.2	1.8	2.4	3.0	3.6	4.8	6.0	7.2
		1.5	.13	.40	.20	1.2	1.6	2.0	2.4	3.2	4.0	4.8
		2.0	.10	.30	.60	.90	1.2	1.5	1.8	2.4	3.0	3.6

Service Factors
1.0 ____ Even Load, No Shock, Infrequent Reversing with Low Starting Torque
1.5 ____ Uneven Load, Moderate Shock, Frequent Reversing with Low Start Torque
2.0 ____ Uneven Load, Heavy Shock, Hi Peak Loads, Frequent Reversals with High Start Torque

3.3.2.1d Sleeve Type Coupling (Geargrip)

A sleeve type coupling consists of two splined hubs with a mating intermediate member of molded neoprene. Because of its construction features, it is capable of normal operation with angular shaft misalignments up to 2°.

Lubrication is not required. All parts are replaceable without disturbing adjacent equipment provided sufficient shaft length is allowed by sliding coupling hubs clear of the sleeve member during disassembly. Select the proper size for your application from Table 5 and follow the selection instructions.

Sleeve Type Coupling Selection Procedure

a. Determine motor characteristics.
b. Determine service conditions.
c. Select the coupling model with an equivalent or slightly greater horsepower than the calculated value in b in Table 5.
d. Turn to Geargrip couplings in the product section and select the specific assembly or individual components in that model number.

TABLE 5 SLEEVE TYPE COUPLINGS SELECTION DATA

Motor Torque	Motor: Normal Torque								Motor: High Torque							
Service	Normal Duty				Severe Duty				Normal Duty				Severe Duty			
Speed, rpm	3500	1750	1160	870	3500	1750	1160	870	3500	1750	1160	870	3500	1750	1160	870
H.P. 1/12	11	11	11	18	11	11	18	18	11	11	18	18	11	18	18	21
1/8	11	11	18	18	11	11	18	21	11	18	18	21	11	18	21	31
1/6	11	18	18	21	11	18	21	21	11	18	21	21	18	21	31	31
1/4	11	18	21	31	18	21	31	31	18	21	31	31	18	31	31	31
1/3	18	21	31	31	18	31	31	31	18	31	31	31	21	31	31	31
1/2	18	31	31	31	21	31	31	31	21	31	31	31	31	31	31	
3/4	21	31	31	31	31	31			31	31			31	31		
1	31	31	31		31	31			31	31			31	31		

Service Conditions

Normal Duty
- speed not exceeding 3600 rpm
- operation less than 10 hours per day
- infrequent stops and starts
- no heavy, pulsating load
- no mechanical or electrical clutch

Severe Duty
- speeds from 3600 to 5000 rpm
- operation runs more than 10 hours per day
- frequent starts and stops
- heavy, pulsating load
- mechanical or electrical clutch

Other types of couplings are also available and are fully described along with technical specifications in the SDP/SI catalogs dealing with couplings [4].

References

[1] Rivin, E.I., *Stiffness and Damping in Mechanical Design*, 1999, Marcel Dekker Inc.

[2] Baranyi, S.J., "The Analytical Design of Universal Joints", Design News, 1969, Sept. 1

[3] Rivin, E.I., "Design and Application Criteria for Connecting Couplings", 1986, ASME Journal of Mechanisms, Transmissions, and Automation in Design, vol. 108, pp. 96-105 (this article is fully reprinted in [1])

[4] Stock Drive Products/Sterling Instrument, *Catalog D790, Handbook of Inch Drive Components* and *Catalog D785, Handbook of Metric Drive Components* or their current catalogs.