TABLE 2 FOURIER EXPANSIONS FOR VIBRATORY PROCESSES IN FIGURE 38 (angles in radians)

Wave Shape Function	Harmonic Amplitude as Fractions of 2h (ω = fundamental frequency)					
Frequency of Harmonics	ω	2ω	3ω	4ω	5ω	6ω
Square wave	$\frac{2}{\pi}$	0	$\frac{2}{3\pi}$	0	$\frac{2}{5\pi}$	0
Saw tooth	$\frac{1}{\pi}$	$\frac{1}{2\pi}$	$\frac{1}{3\pi}$	$\frac{1}{4\pi}$	$\frac{1}{5\pi}$	$\frac{1}{6\pi}$
Repeated steps	$\frac{2\sin \pi\lambda}{\pi}$	$\frac{2\sin 2\pi\lambda}{2\pi}$	$\frac{2\sin 3\pi\lambda}{3\pi}$	$\frac{2\sin 4\pi\lambda}{4\pi}$	$\frac{2\sin 5\pi\lambda}{5\pi}$	$\frac{2\sin 6\pi\lambda}{6\pi}$

To illustrate this approach in a particular case, let's consider a connecting-rod motion of a slider-crank mechanism, Figure 39, as in internal-combustion engines. This motion can be shown to have the following Fourier expansion:

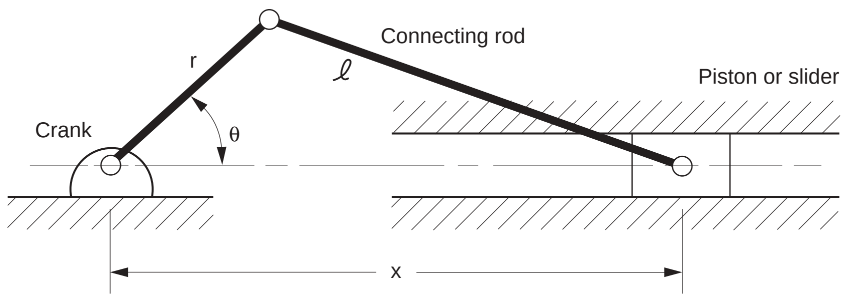

Figure 39 Schematic of a Slider-Crank Mechanism

r = crank length, in.
ℓ = connecting rod length, in.
θ = crank angle, rad or deg.
x = piston placement (piston motion in-line with crank pivot), in.
ω = crank speed, assumed constant, rad/sec
a = piston acceleration, in/sec^2

$$\frac{x}{r} = A_0 + \cos\theta + \frac{1}{4}A_2\cos 2\theta - \frac{1}{16}A_4\cos 4\theta + \frac{1}{36}A_6\cos 6\theta \ldots \qquad (46)$$

$$-\frac{a}{r\omega^2} = \cos\theta + A_2\cos 2\theta - A_4\cos 4\theta + A_6\cos 6\theta \ldots \qquad (47)$$

where A_2, A_4, A_6 are given as follows in Table 3 [4].

TABLE 3 COEFFICIENTS FOR FOURIER EXPANSION OF CONNECTING ROD MOTION

ℓ/r	A_2	A_4	A_6
3.0	0.3431	0.0101	0.0003
3.5	0.2918	0.0062	0.0001
4.0	0.2540	0.0041	0.0001
4.5	0.2250	0.0028	—
5.0	0.2020	0.0021	—

10.0 DESIGN PROBLEM EXAMPLES

The following are a number of problems intended to familiarize the reader with the basic applications of vibration isolators. More advanced techniques which would result in stiffer isolators while achieving adequate isolation can be found in [1].

NOTE: In the following problems, unless otherwise stated, it is assumed that the loads are evenly distributed among the mounting points.

Problem No. 1

A metal tumbling unit weighing 200 lbs and driven by a 950 rpm motor is to be mounted for at least 81% vibration isolation effiecency from the tumbling drum and motor unbalance (one cycle per revolution, or 950 cpm) using 4 cylindrical mounts in shear. Select the isolators.

The weight load per mounting is (1/4) x 200 lbs = 50 lbs. From the basic vibration chart, Figure 12, a forcing frequency of 950 cpm (~ 16Hz) and 81% isolation lead to a point of intersection corresponding to a static deflection of 0.25 in.

Cylindrical mount Part Number V10Z 2-311C, loaded in shear, has a deflection 0.32 in. at 50 lbs. Since this deflection is in excess of 0.25 in., the isolation will be greater than the design minimum. From the basic chart in Figure 12, it is seen to be between 85-90%.

Problem No. 2

Consider the tumbling unit of Problem No. 1 and suppose the motor speed were increased to 2500 rpm. What isolators could be used, allowing loading both in shear and in compression?

From the basic vibration chart, Figure 12, for a forcing frequency of 2500 cpm and 81% isolation, we find a static deflection of about 0.037 in. Hence we must look for isolators with a load rating not less than 50 lbs and with a corresponding deflection of not less than 0.037 in. The following mounts can be considered:

Load in Compression	Load in Shear
V10Z 2-300C (0.078 in. deflection)	V10Z 2-330B (0.14 in. deflection)
V10Z 2-317C (0.078 in.)	V10Z 2-311C (0.31 in.)
V10Z 2-310B (0.138 in.)	
V10Z 2-314C (0.042 in.)	

Amongst these, the highest percentage of isolation is afforded by the mount with the largest deflection (V10Z 2-311C), provided that such a deflection is permissible.

Problem No. 3

A small business machine is to be mounted for 81% vibration isolation effiecency. The weight is 25 lbs and there are 4 mounting points. What additional information is required for the selection of the vibration isolation system?

Information which is needed is as follows: allowable vibration amplitudes of the machine, as a function of frequency; frequency of disturbing force; direction and point of application of disturbing force; space limitations, if any; ambient conditions, if unusual; mass and compliance distribution of machine – if not uniform.

Problem No. 4

A device contains 4 symmetrically located special-configuration isolators (Finger-Flex), Part Number V10R 4-1502D, each isolator deflecting just over 0.07 in. at 20 lb load. In order to obtain satisfactory vibration isolation, it is desired to increase the deflection from 0.07 in. to 0.14 in., the load remaining the same. How can this be done?

One way is to stack two (identical) mounts in series, see section 7.2, each of the four isolators being replaced by such a set.

Problem No. 5

A unit which is to be mounted for 81% vibration isolation efficiency has a forcing frequency of 1500 cpm (25 Hz), weighs 1080 lbs and is to use 6 vibration isolators in shear. Isolators with a female tap are required. Select an isolator model.

The load per isolator is 1080/6 = 180 lbs. At 1500 cpm and 81% isolation effiecency, the basic vibration chart, Figure 12, gives a static deflection of 0.10 in.

Isolator V10Z 2-308C loaded in shear has a deflection of about 0.13 in. at 180 lbs. This being in excess of 0.10 in., the degree of isolation is certainly satisfactory. This model has a female tap.

Problem No. 6

A 275 lb motor is mounted with cylindrical isolators V10Z 2-311C loaded in shear at six points, the forcing frequency being 1100 cpm (~ 18 Hz). What is the percentage of vibration isolation attained?

The load per isolator = 275/6 = 45.8 lbs, assuming mounts to be symmetrically located, so that load is evenly distributed. From the design information furnished in the catalog, the shear deflection of the isolator at this load is ~ 0.28 in.

From Figure 12, the point of intersection of 0.28 in. static deflection and forcing frequency of 1100 cpm gives an isolation efficiency of about 87%.

Problem No. 7

An air conditioner weighs 250 lbs and is driven by a motor at 1700 rpm. The unit is mounted in shear on four V10Z 2-317B cylindrical isolators. Is this design satisfactory?

The isolated unit is not properly installed because the maximum load rating for this isolator, as indicated in the catalog, is 21 lbs in shear and 40 lbs in compression. The load per mount is 250/4 = 62.5 lbs. Even if the isolator is installed so that it is loaded in compression, it would not be satisfactory, since the load (62.5 lbs) is significantly in excess of the 40 lbs recommended limit.

Mounts, which have sufficient load capacity, are as follows (with static deflection indicated):

Part Number	Static Deflection
V10Z 2-310B (Compression)	(0.175 in.)
V10Z 2-311C (Shear – marginal)	(0.38 in.)
V10Z 2-330B (Shear)	(0.175 in.)

The choice of isolators depends (amongst other matters) on the degree of isolation desired. With any of the above isolators, this will be in excess of 81% for the forcing frequency equal to motor rpm.

Problem No. 8

If, in the preceding problem, the air conditioner weighs 350 lbs, what is the choice of mounts?

The load/mount is 350/4 = 87.5 lbs. The following mounts can be considered (with static deflection indicated):

Part Number	Static Deflection
V10Z 2-314C (Compression)	(0.075 in.)
V10Z 2-311D (Compression)	(0.094 in.)
V10Z 2-330B (Shear)	(0.26 in.)

At 1750 cpm, 81% vibration isolation corresponds to a static deflection of 0.074 in.

Problem No. 9

A computer weighs 200 lbs. It is to be vibration isolated with 4 mounts. The forcing frequency is 1750 cpm (~ 29 Hz). If the isolators are to be loaded in compression, what models are available and what is the percentage of vibration isolation attained in each case?

The load per mount is 200/4 = 50 lbs. Hence, isolators with a load capacity of at least 50 lbs in compression are needed. For each isolator, the catalog contains data (table or plots) from which static deflection under a 50 lb load can be found. From the basic vibration chart, Figure 12, with this value of static deflection and a forcing frequency of 1750 cpm, the point of intersection defines the attained vibration isolation efficiency. Thus, the following isolators can be selected:

Type of Mount	Part Number	Static deflection, in., at 50 lb compression	Isolation Efficiency, %
Cylindrical	V10Z 2-317C	0.078 in.	82%
Cylindrical	V10Z 2-300C	0.078 in.	82%
Cylindrical	V10Z 2-310B	0.138 in.	91%
Special (Finger-Flex)	V10R 4-1506B	0.14 in.	~ 91%
Special (Finger-Flex)	V10R 4-1506C	0.09 in.	~ 85%

Problem No. 10

A 4-cylinder engine weighing 370 lbs and operating at 2800 rpm is to be isolated for 81% vibration isolation for one-per-revolution excitation frequency. Discuss the possible selection of isolators.

The lowest frequency to be isolated is 2800 cpm (~ 46.5 Hz). In general, it is desirable to arrange the mounts so that the resultant of the loads, supported by the mounts, passes through the C.G. This is the same condition (but stated differently) as the one described in Section 5.0 above. If the isolators are symmetrically arranged, and each isolator carries the same load, this usually means that the symmetry axis of the isolators passes through the C.G. In this case, we are concerned not only with the translational displacement of the engine as a whole, but also with engine rotation. In addition, flexible gas lines and the throttle linkage can vibrate and their vibration isolation may pose an additional problem.

At 2800 cpm and 81% isolation efficiency, the basic vibration chart, Figure 12, gives a static deflection of about 0.03 in. The load is 370/4 = 92.5 lbs per mount.

Consider rectangular mount V10Z 6-500B loaded in shear. This has a deflection of about 0.12 in. in shear, which can accommodate the rotation of the engine about the torque-roll axis. The mount deflection in compression would serve to accommodate the shock load in translation.

Problem No. 11

An 80 lb fan is to be vibration isolated in shear at four points with at least 93% vibration isolation efficiency when the fan is turning at 2000 rpm. Specify the mounts.

The main source of vibration is rotor unbalance, and the transmission to ground of the vertical component of this force, (which is sinusoidal) is undesirable (see [1] for isolation of other vibration components). Hence, consider Section 3.1, Equation (8), with negligible damping.

Solution #1:

From the basic vibration chart, Figure 12, 93% isolation at a forced frequency of 2000 cycles/minute (~ 33.3 Hz) corresponds to a static deflection of about 0.14 in. and to a natural frequency of about 500 cpm (~ 8.5 Hz).

Consider cylindrical vibration isolators, Part Number V10Z 2-300C loaded in shear, which deflects about 0.17 in. at 20 lbs and appears to be suitable for this application.

Solution #2: (Analytical)

When the isolation efficiency is 93%, the force transmissibility, μ_F is 1 – 0.93 = 0.07 or 7%. With zero damping (δ = 0), Equation (8) gives for $\delta = 0$:

$$\mu_F = \frac{1}{\pm\left(1 - \frac{f^2}{f_n^2}\right)} \qquad (48)$$

where $\mu_F = 0.07$;
"+" is to be used when $f < f_n$, and
"–" is to be used when $f > f_n$.

Since for good isolation, $f > f_n$, "–" sign will be used.

Solving for f/f_n from Equation (48), we obtain $f/f_n = 3.91$.
Since f = 33.3 Hz, $f_n = 8.52$ Hz.
From Equation (4), solving for $x_{st} = 0.344$ cm = 0.136 in.

These calculations agree adequately with the values found from the chart in Figure 12.

Problem No. 12

Data as in problem 11, but damping is estimated at $c/c_{cr} = 0.1$, see Section 2.4 above. How would it change the specifications?

The force transmissibility, μ_F, corresponding to 93% vibration isolation efficiency, is 0.07 and the forcing frequency is 2000 cpm (33.3 Hz). From Figure 10, for damping ratio $c/c_{cr} = 0.1$ at $\mu = 0.07$, the frequency f/f_n = ~ 5.

Hence, $f_n = 2000/5 = 400$ cpm (~ 6.7 Hz).

From the basic chart, Figure 12, this natural frequency corresponds to a static deflection of ~ 0.21 in. Since the load remains at 20 lbs per mount, the isolators specified for Problem 11 are too stiff. Isolator V10Z 2-310B loaded in shear appears to be satisfactory (deflection ~ 0.33 in. at 20 lbs).

This problem could also have been solved by a computer program, or analytically. In the latter case, Equation (8) can be solved for f_n at the value $c/c_{cr} = 0.1$, f = 33.3 Hz.

Comparison of Problems 11 and 12 shows that viscous damping in isolators results in increasing transmissibility at the isolation frequency range (which starts from $f/f_n = \sqrt{2} = 1.41$); i.e., reducing effectiveness of isolation and requiring softer isolators to get the desired efficiency. This is the price to pay for very desirable reduction of resonance amplitudes. When the damping is not viscous but material damping, such as in isolators with rubber flexible elements, the deterioration of the high frequency isolation is minimal.

Problem No. 13 A Vibroactive Object (Machine)

A small machine tool weighs between 3.5 lbs and 5 lbs depending on the weight of the work piece. When the forcing frequency, which is generated by the vibration source inside the machine, is between 60-90 Hz. and again when it is within 200-400 Hz range, the vibration is objectionable. Design a vibration mount for a 3-point support with vibration isolation efficiency of not less than 81%.

In the absence of more information, we may assume that isolators have zero damping. If we isolate for the lowest objectionable forced frequency (60 Hz), that would take care of all the troublesome regions.

From the basic vibration chart, Figure 12, an 81% isolation ratio at a forced frequency of 60 Hz corresponds to a static deflection of about 0.019 in. The weight supported by each mounting ranges from 3.5/3 lbs to 5/3 lbs, or from 1.17 to 1.67 lbs. The natural frequency is read off from the chart at about 23 Hz. Hence, the vibration mount specification is:

0.019 in. deflection

1.17 lbs to 1.67 lbs supported weight.

Square mount V10Z 1-321B loaded in compression is a possibility. Considering the special configuration (Finger-Flex) mounts, Part Number V10R 4-1500A can be selected. Its deflection at 1.17 lbs is only about 0.03 in. In view of its construction, the spring rate of this mount increases rapidly with deflection and the special configuration unit would be both more economical in the use of space and more effective in taking care of overloads, if this should arise.

Problem No. 14 A Vibration/Shock Sensitive Object

Sensitive radio equipment is to be mounted with a 3-point suspension on a boat. Protection from engine disturbance is required, as well as from impacts of waves and from bumping against pier. The equipment weighs 54 lbs and the engine runs at 2000 rpm.

Here we have both steady vibrations at 2000 cycles/min as well as shock loads, caused by wave pounding and by bumping against the dock. We have no precise information on the latter and need to do the best we can.

For the steady vibration, consider Equation (8) with zero damping which becomes Equation (48). At 81% efficiency and forcing frequency of 2000 cycles/min, the basic vibration chart, Figure 12, gives a static deflection of about 0.058 in. The load per mount is 54/3 = 18 lbs. The natural frequency obtained from the chart is about 760 cycles/min = 12.7 Hz.

V10R 4-1504B ring-style special-configuration (Finger-Flex) mount approximately fulfills this condition.

In order to limit the effect of shock loads, conical bumpers may be added to limit the horizontal shock load, possibly with the V10Z 7-1020C type.

It can, however, also be made an arbitrary guess and assumption that the pier and waves effects are equivalent approximately to a 0.5 mph sudden change of horizontal velocity of the boat and try to design the vibration mount for this condition. This will provide some insight into how much of a sudden velocity can be expected to be cushioned by vibration mounting. This corresponds to Section 3.3.1 of horizontal motion and negligible damping ($c/c_{cr} = 0$).

It is also important to know how much force the sensitive radio equipment can take without damage. Often such a force is expressed as a g-load; i.e., how many times its own weight the equipment can survive. For example, a 1/2 g-load means that the object can withstand a maximum force of (1/2) (54) = 27 lbs without damage. Usually, the allowable shock loads are determined by testing. Let's assume that the maximum safe load on the radio equipment is 1g or 54 lbs.

From Equation (15), Section 3.3.1, we have

$$\frac{a_{max}}{g} = \frac{2\pi fV}{g} = 1,$$

where V = 0.5 mph = 8.8 in/sec = 0.22 m/sec. Hence, f = 7 Hz.

This frequency is quite low, and associated with undesirably large deflections of vibration isolators. This suggests using a cylindrical mount loaded in compression for the vertical (engine) vibrations and having reasonably large compliance in the horizontal (shear) mode to take care of some of the shock, with a conical bumper to limit excessive horizontal deflections.

For example, cylindrical mount V10Z 2-300A has 0.075 in. deflection at 20 lbs compressive load, while in shear, the deflection at 16 lbs is about 0.32 in., or six times as much. This is an overload, but might still be considered due to the infrequent occurences of the shock load.

The natural frequency in the shear mode based on the 16 lb load is about 5.7 Hz, which is 20% lower than the 7 Hz specified above.

From Equation (18), $\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = 1$, thus d_{max} = 0.32 in. Note that d_{max} is computed as if the weight were supported in shear.

This is too large a maximum deflection. A conical bumper should be used to limit the deflection by 0.20 in., say. Alternatively, a stronger and stiffer mount should be considered, for example, V10Z 2-300B, which deflects 0.26 in. at 18 lbs in shear. The isolation effectiveness in compression is reduced to about 65%; and while the isolation ratio in shear is also reduced, so is the corresponding maximum deflection. In addition, the conical bumpers should be added. The final choice of mounts is a matter of judgment.

Problem No. 15

A single-cylinder gasoline engine drives a one-cylinder air compressor with belt. Both units are bolted to a light-gage metal pan, which is welded to the top of an air-receiver tank, which is in turn mounted to a four-wheel steel-tired dolly. The whole unit vibrates and walks all over the floor. The engine weighs 100 lbs and turns at 3000 rpm. The compressor weighs 120 lbs and turns at 1200 rpm. The tank weighs 25 lbs and the dolly weighs 50 lbs. What can be done?

Possibly good rubber tires on the dolly and/or wheel suspension would help. If the tank is mounted to the dolly, total weight is:

W = 100 + 120 + 75 = 295 lbs.

The lowest-frequency disturbing force is that due to the air compressor; i.e., 1200 cycles/min = 20 Hz. At 81% vibration isolation efficiency, Figure 12 gives a static deflection of the isolator of about 0.15 in. Considering a 4-mount suspension, the load per mount is 74 lbs.

Cylindrical mount V10Z 2-310B would be a possibility, loaded in compression. If the dolly continues to move, since it weighs only 50 lbs, it might require a little softer material than the 40-durometer rubber, in order to effect more isolation.

Next, consider mounting on isolators the pan that holds the engine and air-compressor unit. The total weight here is 100 + 220 lbs and with the same static deflection of 0.15 in., a V10Z 2-310A mount would suffice in compression, considering the fact that the chart shows the V10Z 2-310B mounts to deflect 0.12 in. at 55 lbs. The lower-durometer mount (Type A, at 30-durometer) should, therefore, approximate the 0.15 in. required deflection. Note that the last letter in the mount identification specifies the approximate durometer hardness of the rubber (A = 30, B = 40, C = 50).

T1 Technical Section

Problem No. 16 Isolation of a Punch Press (also see [1]).

This is one of the most difficult applications for isolation. Shock absorption is all that can be expected. Unit weighs 1500 lbs, sits on four feet, operates at 50-100 rpm, and is driven by a 5 H.P., 1750 rpm electric motor, the flywheel turning at 250 rpm.

While many vibration problems deal with sinusoidal or nearly sinusoidal forces and some (such as in package cushioning) deal with essentially sudden velocity changes, here we have a suddenly applied force, which is periodic, but not harmonic. The force-time variation is essentially that of the "Repeated Step" in Section 9.0.

If we assume that the punching operation of the press occurs, say, during 30° of crank rotation, then the λ in this case (Repeated Step, Section 9.0) is 30/360 = 1/12 = 0.08333. From Section 9.0, we find that the amplitude of the fundamental harmonic is $(2/\pi)\sin\pi\lambda$ or 0.164. This is only about 16% of the amplitude of the force pulse, and its frequency is operating frequency (50-100 rpm or 0.85 - 1.7 Hz).

Consider, however, the 4th harmonic (200-400 cycles/min). Its amplitude is $(2\sin 4\pi\lambda)/4\pi$ = .1376 or 13.8%. This is not much less than the amplitude of the basic (fundamental) frequency. This shows that in the punch-press type of disturbing force, the higher harmonics cannot be neglected.

The fundamental frequency (50-100 cycles/min) is so low that isolation with vibration isolation mounts would lead to their excessive static deflections. However, it is conceivable that a practical vibration isolator would be successful in isolating some of the significant higher harmonics. For vibration isolation of punch presses, the following few rules might be useful (also see [1]).

1. Slow-speed presses should be mounted with mounts of greater deflection than high-speed presses.
2. Mount deflections used for presses by direct installation of vibration isolation mounts under their feet may vary from 1/32 in. to 3/4 in. depending largely on operating speed and stroke length, with the smaller deflection being the more common.
3. There may be several static deflections that will work, while other static deflections interspaced in between them will not work; i.e., 1/16 in. and 3/16 in. may work, while 1/8 in. may not work. This can be caused, at least in part, by the fact that a significant set of higher harmonics may be isolated at one deflection, but not at another.
4. Even the best mounting system will still transmit a significant amount of vibration and shock.
5. If the ultimate in isolation is required, the punch press must be attached solidly to an inertia block of large mass and the entire press and the block mounted on vibration isolators.

Problem No. 17

A relatively high-precision experiment is to be conducted in the laboratory of a textile plant. The laboratory floor vibrates at an amplitude of 0.0005 in. due to the operation of industrial sewing machines and other textile machinery. The basic floor-vibration frequencies are that excited by the industrial sewing machines, which operate in the 1500-5000 rpm range. It is desired to vibration isolate the test unit, which weighs 25 lbs, with a four-point mounting at not less than 81% isolation of displacement.

At 81% displacement isolation, the displacement transmissibility, μ_x is 0.19. It is calculated using the same equations (8) and (48) as for μ_F.

For zero damping, Equation (48) gives:

$$\mu_F = \frac{1}{\pm\left(1 - \dfrac{f^2}{f^2_n}\right)}$$

Taking f as the lowest sewing-machine speed (1500 cycles/min or 25 Hz) and $\mu_F = 0.19$, we find f_n = 600 cycles/min = 10 Hz. The static deflection of the vibration isolators is determined from Equation (4), as x_{st} = 0.25 cm = 0.1 in. The same result can be obtained from Figure 12. The isolation specification, therefore, is 0.10 in. static deflection at a load of 25/4 = 6.25 lbs.

Considering cylindrical vibration isolators, mount V10Z 2-316B loaded in shear, has a 0.10 in. deflection at about 6.25 lbs. Soft mounts, such as this one, are often using shear deformation of the flexible elements.

Problem No. 18

Data as in Problem 17, except that system damping is estimated at 10% of critical (δ = ~ 0.63). Reevaluate the specification of the isolators.

In Problem 17, we found that the displacement transmissibility corresponding to 81% isolation is $\mu_x = 0.19$; and that the lowest forcing frequency, f = 1500 cycles/min. = 10 Hz. From Figure 10, p.T1-11, which applies to μ_x as well as to μ_F, we find that the given value of the transmissibility at δ = ~ 0.63 yields a frequency ratio f/f_n = ~ 2.7. Hence, f_n = 1500/2.7 = ~ 9 Hz.

At this natural frequency, the basic vibration chart (Figure 12) gives a static deflection of about 0.117 in. The load per mount, as in Problem 17, is 6.25 lbs.

The isolator specification V10Z 2-316B of Problem 17 remains satisfactory.

Problem No. 19

An impact testing machine consists of a simple pendulum of length 4 feet and weight 5 lbs, which is initially horizontal. It is released and at the bottom of its swing impacts the test object. In this test, it comes to rest essentially instantaneously (inelastic impact). The object (equipment to be tested) weighs 100 lbs and is capable of withstanding accelerations up to 2g. Design a vibration isolation/mounting system so that the equipment will survive the impact test.

The velocity aquired by the pendulum in the 4 foot drop is

$V_o = \sqrt{2gh}$, = 193 in/sec (striking velocity), where g = 386 in/sec^2; h = 4 ft. x 12 = 48 in.

The momentum of the pendulum just prior to impact is equal to the impulse "I" applied to the object. It is equal to the mass of the pendulum times its velocity,

$$I = \frac{5}{386} \times 193, \text{ or } 2.5 \text{ lb-sec.}$$

If the pendulum retains a residual velocity V_p' just after striking the test object, "I" would be computed from

$$I = (V_p - V_p') \times (\text{mass of pendulum}).$$

The impact result is an essentially sudden velocity change by V_1, of the equipment, which, can be calculated from Equation (20) as:

$$V = \frac{Ig}{W} \text{ in/sec.}$$

$$= \frac{(2.5)\,(386)}{100} \text{ in./sec.} = 9.65 \text{ in./sec.}$$

This value of V can be used in Equation (15), or

$$\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = \frac{2\pi f V}{g}$$

with a_{max} = 2g and V = 9.65 in./sec. Then $f_n = g/\pi V$ = 1.35 Hz.

Realization of such low natural frequency (albeit, in a horizontal direction; less destabalizing than in the vertical direction) is a very special problem. It can be addressed by utilizing information in [1].

Problem No. 20 Vibration Isolation of High Precision Object

Formulate requirements for vibration isolation system (f_n and δ) for a projection aligner for semiconductor manufacturing for two conditions:

A - the apparatus is installed on the floor of a regular manufacturing plant so that for vertical direction $X_f(f)$ = const = 3.0 μm for frequencies 3 ~ 30 Hz and $X_f(f) = 3.0\,\frac{30}{f}$ μm for frequencies f > 30 Hz; for the horizontal direction $X_f(f)$ = const = 2.5 μm for frequencies 2 ~ 20 Hz, and $X_f(f) = 2.5\,\frac{20}{f}$ μm for frequencies f > 20 Hz.

B - floor vibration levels corresponding to line VC-B in Figure 15 (both for vertical and horizontal directions).

Vibration sensitivity of this apparatus to vertical and horizontal vibration of its frame (base) was experimentally determined and shown in Figure 40. These plots show what amplitude of vibration X_b at the given frequency results in a relative vibration amplitude in the working zone (image motion) not exceeding the tolerated amplitude Δ_o = 0.1 μm. The minima on these plots represent structural natural frequencies of the devices. At each frequency f, transmissibility from the base to the work zone is $\mu_f = \Delta_o/X_b$.

Since the vibration sensitivity μ_f of this precision object is known (can be easily calculated from the experimentally obtained plots in Figure 40) then Expression (12b) can be used for specifying vibration isolation parameters.

Table 4 gives the values of μ_f (Δ_o divided by the ordinate of the plot in Figure 40 for a given frequency) calculated for critical points from the plots in Figure 40 for vertical and horizontal directions, respectively.

Table 4 also contains values of Φ_{Av} and Φ_{Ah} calculated for these points using Equation (12b) and vertical and horizontal floor vibration amplitudes specified in A.

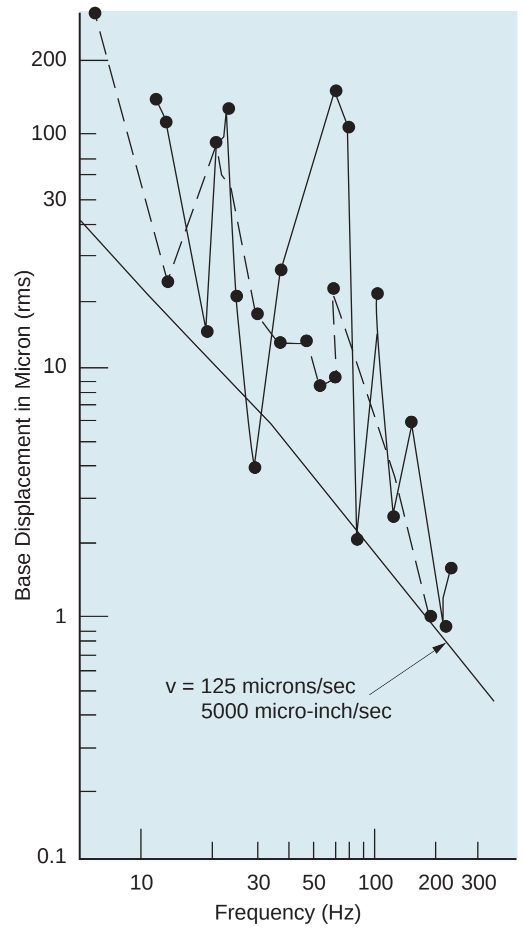

Figure 40 Vibration Sensitivity for Projection Aligner

Perkin-Elmer Microlign Mod. 341 for 0.1 μm Image Motion (Solid Line - Limit of Vertical Floor Vibration Amplitude, Broken Line - Limit of Horizontal Floor Vibration Amplitude).

TABLE 4 VIBRATION ISOLATION SYNTHESIS FOR FIGURE 40

A. Vertical Direction (Y-axis)			
f Hz	μ **(f)**	Φ_{Av} Hz	Φ_{Bv} Hz
11	0.0083	4.51	12.9
12	0.010	12.3	36.6
20	0.087	7.0	26.9
25	0.0091	26.9	116
30	0.056	13.0	61
32	0.303	6.3	29.7
41	0.05	22.5	106
70	0.0077	128	601

B. Horizontal Direction (X-axis)			
f Hz	μ **(f)**	Φ_{Ah} Hz	Φ_{Bh} Hz
7	0.0033	13.7	23.1
12	0.05	6.05	37.5
22	0.125	22.3	78
65	0.071	49.6	174
70	0.090	49.2	172
100	0.090	84	294

The values of Φ_{Bv} and Φ_{Bh} were calculated using floor vibration levels corresponding to line VC-B in Figure 15 (both for vertical and horizontal directions). Since plots in Figure 15 are given for vibratory velocity V_f, vibration displacement amplitudes X_f were calculated for each frequency of interest as $X_f = V_f/2\pi f$.

Values of Φ_A calculated per Specification A are interesting only for comparison, since high precision microelectronic production equipment is never used in conventional plant facilities, only in specially designed buildings complying with some of VC criteria.

It can be seen from Table 4A that the lowest value of Φ_{Av} (case A) for vertical direction is 4.51 Hz. If vibration isolators with medium damping $\delta_v = 0.6$ are used, then from Equation (12a) the required vertical natural frequency $f_v = 4.51\sqrt{0.6} = 3.04$ Hz. However, if isolators made of rubber with high damping $\delta_v = 1.2$ are used, then $f_v = 4.51\sqrt{1.2} = 5.0$ Hz, which can be realized by passive isolators with soft rubber flexible elements.

Much stiffer isolators ($f_{vz} > 14$ Hz) can be used to comply with values of Φ_{Bv}, per Specification B, which represent (according to not very stringent requirement VC-B) floor conditions at the microelectronics industry installations.

A similar situation is seen in Table 4B; however, realization of natural frequencies corresponding to Φ_{Bh} (4.7 Hz for $\delta_v = 0.6$, 6.63 Hz for $\delta_v = 1.2$) in horizontal directions with elastomeric isolators does not present any difficulty; even much lower values can be easily realized.

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