3. The displacements (max. deflections) of the mounts can be calculated from Equation (45) for any given single disturbing force or torque. If several force/torques act simultaneously, vector addition of forces in different directions is required, and Equation (45) cannot be used.
4. The case of a horizontal disturbing force has not been considered in this presentation.
5. Other things being equal, the best arrangement for the mounts is to arrange them so that their resultant force passes through the center of gravity of the equipment and that its line of action is a principal axis. If there is a resultant torque about the center of gravity, its direction should be about a principal axis through the center of gravity. However, if this arrangement is impractical, it need not be adhered to.

9.0 COMPLEX DRIVING FORCES

When the disturbing forces are neither sinusoidal nor suddenly applied, the vibration analysis becomes more complicated. While it is more difficult to give general guidelines or methods of analysis, one can consider every force-time variation as composed of components of different frequencies, each being a multiple of the basic (usually driving) frequency. Mathematically, this is known as expanding an arbitrary function into a Fourier series. Once these frequency components (harmonics) are determined, each one being sinusoidal at a different frequency, any component can be analyzed like a sinusoidal force. This can provide at least some understanding of the vibration phenomenon. Often the lowest-frequency (fundamental) component predominates and is the most important component to analyze. It is possible, however, that the design of the vibration isolation system will appear unfeasible on the basis of an analysis of only the fundamental component, whereas the exact analysis would show that a vibration isolation mounting can be useful; i.e., sometimes an analysis of components of several frequencies may be required [1]. This, however, may be quite difficult. In such cases, resolving an arbitrary force-time variation into several harmonics can provide some insight.

The following represents data in the Fourier series (decomposition into several harmonics) of some representative force-time variations in Figure 38, which are neither sinusoidal nor sudden. Each force is assumed to be a periodic function of the time;

$\lambda = \tau/T$, where τ is pulse width, T is the process period;
ω = fundamental frequency.

The Fourier expansions for these forcing functions are given in Table 2.

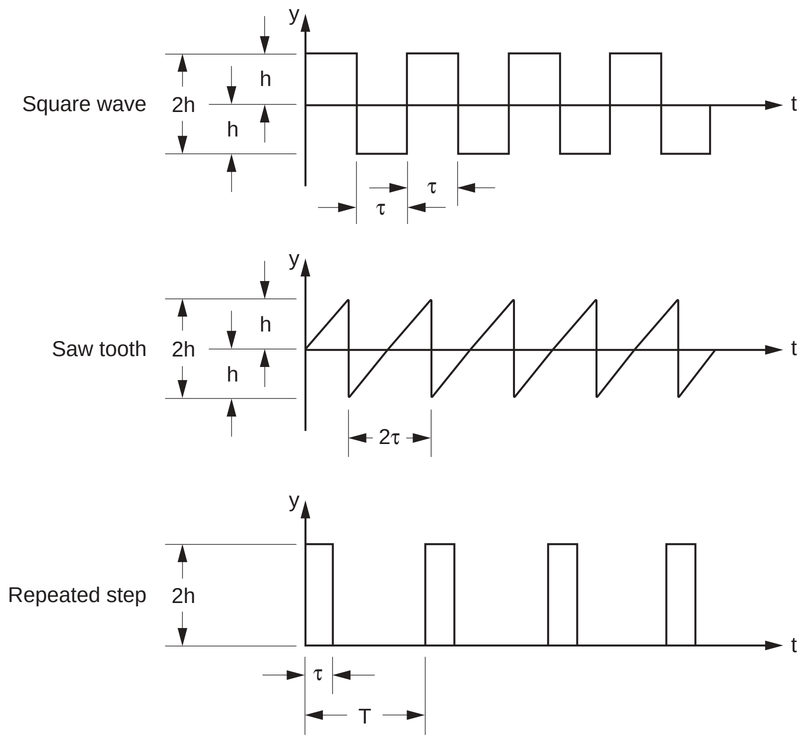

Figure 38 Typical Periodic Nonsinusoidal Vibratory Processes

T1 Technical Section

TABLE 2 FOURIER EXPANSIONS FOR VIBRATORY PROCESSES IN FIGURE 38 (angles in radians)

Wave Shape Function	Harmonic Amplitude as Fractions of 2h (ω = fundamental frequency)					
Frequency of Harmonics	ω	2ω	3ω	4ω	5ω	6ω
Square wave	$\frac{2}{\pi}$	0	$\frac{2}{3\pi}$	0	$\frac{2}{5\pi}$	0
Saw tooth	$\frac{1}{\pi}$	$\frac{1}{2\pi}$	$\frac{1}{3\pi}$	$\frac{1}{4\pi}$	$\frac{1}{5\pi}$	$\frac{1}{6\pi}$
Repeated steps	$\frac{2\sin \pi\lambda}{\pi}$	$\frac{2\sin 2\pi\lambda}{2\pi}$	$\frac{2\sin 3\pi\lambda}{3\pi}$	$\frac{2\sin 4\pi\lambda}{4\pi}$	$\frac{2\sin 5\pi\lambda}{5\pi}$	$\frac{2\sin 6\pi\lambda}{6\pi}$

To illustrate this approach in a particular case, let's consider a connecting-rod motion of a slider-crank mechanism, Figure 39, as in internal-combustion engines. This motion can be shown to have the following Fourier expansion:

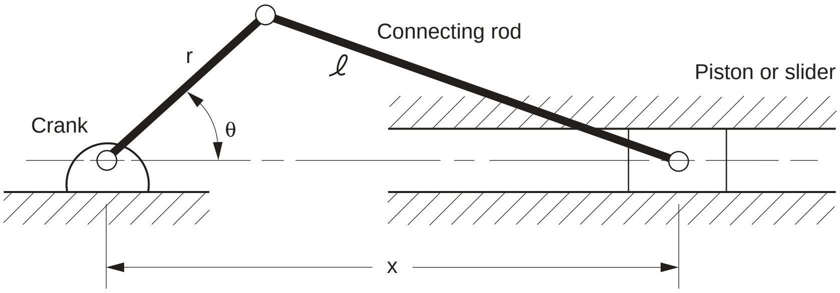

Figure 39 Schematic of a Slider-Crank Mechanism

r = crank length, in.
ℓ = connecting rod length, in.
θ = crank angle, rad or deg.
x = piston placement (piston motion in-line with crank pivot), in.
ω = crank speed, assumed constant, rad/sec
a = piston acceleration, in/sec^2

$$\frac{x}{r} = A_0 + \cos\theta + \frac{1}{4} A_2 \cos 2\theta - \frac{1}{16} A_4 \cos 4\theta + \frac{1}{36} A_6 \cos 6\theta \ldots \quad (46)$$

$$-\frac{a}{r\omega^2} = \cos\theta + A_2 \cos 2\theta - A_4 \cos 4\theta + A_6 \cos 6\theta \ldots \quad (47)$$

where A_2, A_4, A_6 are given as follows in Table 3 [4].

TABLE 3 COEFFICIENTS FOR FOURIER EXPANSION OF CONNECTING ROD MOTION

ℓ/r	A_2	A_4	A_6
3.0	0.3431	0.0101	0.0003
3.5	0.2918	0.0062	0.0001
4.0	0.2540	0.0041	0.0001
4.5	0.2250	0.0028	—
5.0	0.2020	0.0021	—

10.0 DESIGN PROBLEM EXAMPLES

The following are a number of problems intended to familiarize the reader with the basic applications of vibration isolators. More advanced techniques which would result in stiffer isolators while achieving adequate isolation can be found in [1].

NOTE: In the following problems, unless otherwise stated, it is assumed that the loads are evenly distributed among the mounting points.