8.0 3-D OBJECT DRIVEN BY VIBRATORY FORCE AND TORQUES

Figure 37 shows an object with its C.G. at C, mounted on 4 flexible mounts and acted upon by a disturbing harmonic force F_y in the y-direction (vertical) and/or by torques, T_x, T_y and T_z acting singly or in combination about the x, y and z axes, which are principal inertia axes passing through the C.G. (point C).

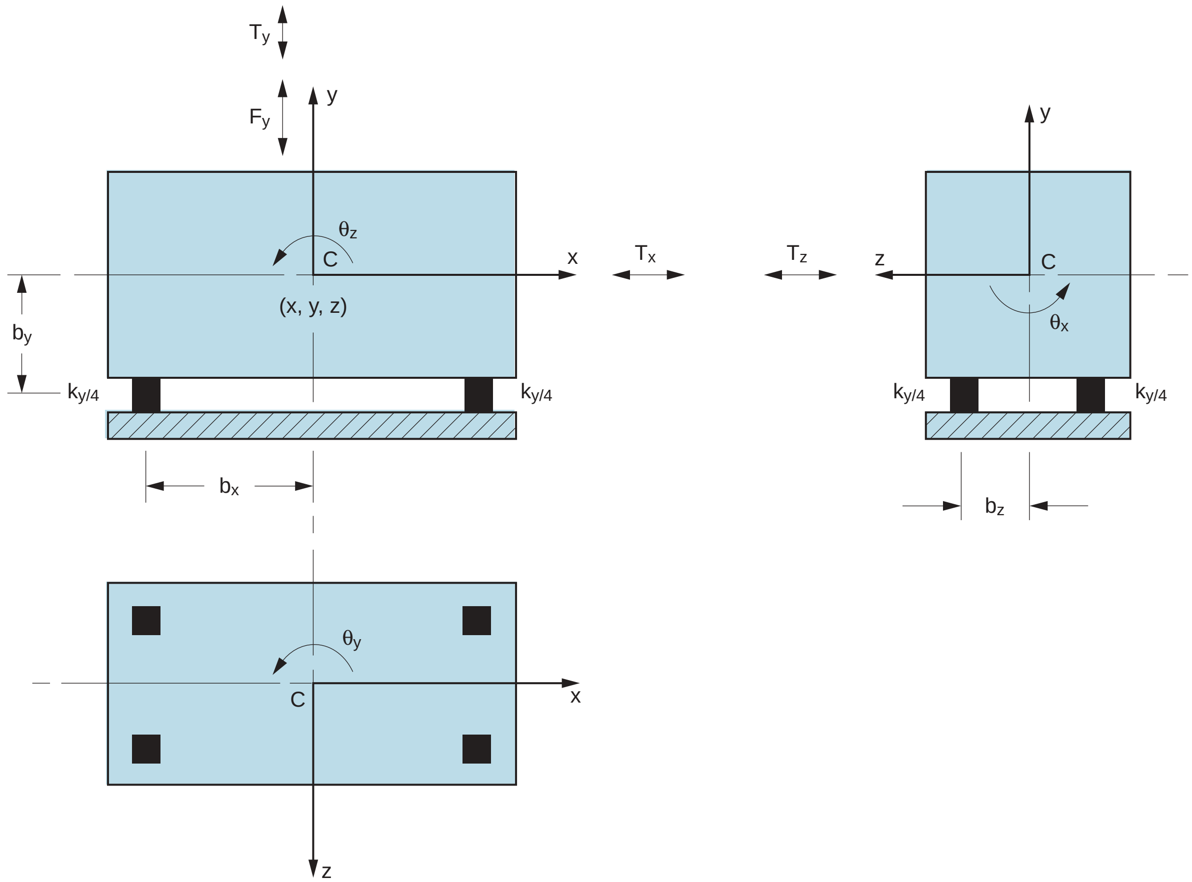

Figure 37 Solid Body on Vibration Isolators

The four mounts are symmetrically disposed relative to the C.G., their location defined by distances b_x, b_y and b_z from the axes, as shown. The mass moments of inertia about the principal inertia axes are I_x, I_y and I_z, respectively. As a result of the external force and torques, the object motion is (a) a displacement of C.G., maximum values of which are denoted by translational motions of the C.G. (x, y, z) and (b) rotations of the object (from equilibrium) about the coordinate axes (θ_x, θ_y, θ_z). These displacements are generally small relative to the major dimensions of the object.

Let: M = mass of object (W/g where W is weight of the object, g = 386 in/sec^2 = 9.8 m/sec^2);

k_y = total vertical stiffness of the four supports in lb./in. or N/m; i.e., 4 times the stiffness of each support if all four supports are identical

k_s = total horizontal or shear stiffness of the four supports; i.e., 4 times the horizontal stiffness of each support, if all supports are identical and for each support $k_x = k_z = k_s$, lb./in. or N/m;

ω = angular frequency of sinusoidally applied force and torques (rad/sec)

Damping is assumed to be negligible.

8.1 Displacement of the Object

Due to F_y only:

$$y = \frac{F_y}{k_y - M\omega^2} \qquad (31)$$

Due to T_z only:

$$x = \frac{T_z b_y k_s}{I_z M\omega^4 - \omega^2 (I_z k_s + k_y b_x^2 M + k_s b_y^2 M) + k_y k_s b_x^2} \qquad (32)$$

$$\theta_z = \frac{T_z (k_s - M\omega^2)}{I_z M\omega^4 - \omega^2 (I_z k_s + k_y b_x^2 M + k_s b_y^2 M) + k_y k_s b_x^2} \qquad (33)$$

Due to T_x only:

$$z = \frac{T_x b_y k_s}{I_x M\omega^4 - \omega^2 (I_x k_s + M k_y b_z^2 + M b_y^2 k_s) + k_y k_s b_z^2} \quad (34)$$

$$\theta_x = \frac{T_x(k_s - M\omega^2)}{I_x M\omega^4 - \omega^2 (I_x k_s + M k_y b_z^2 + M b_y^2 k_s) + k_y k_s b_z^2} \quad (35)$$

Due to T_y only:

$$\theta_y = \frac{T_y}{k_s (b_x^2 + b_z^2) - I_y\omega^2} \quad (36)$$

In these equations F_y, T_x, T_y and T_z represent peak values of the corresponding applied force or torques.

8.2 Undamped Natural Frequencies

Source	Mode	Equation	
F_y	Translation along y-axis	$\omega_1 = \sqrt{\frac{k_y}{M}}$	(37)
T_z	Rotation about axes parallel to z-axis	$\omega_2 = \sqrt{A - \sqrt{A^2 - \frac{k_y k_s b_x^2}{I_z M}}}$	(38)
		$\omega_3 = \sqrt{A + \sqrt{A^2 - \frac{k_y k_s b_x^2}{I_z M}}}$	(39)
		where $A = \frac{k_s}{2M} + \frac{k_y b_x^2 + k_s b_y^2}{2I_z}$	(40)
T_x	Rotation about axes parallel to x-axis	$\omega_4 = \sqrt{B - \sqrt{B^2 - \frac{k_y k_s b_z^2}{I_x M}}}$	(41)
		$\omega_5 = \sqrt{B + \sqrt{B^2 - \frac{k_y k_s b_z^2}{I_x M}}}$	(42)
		where $B = \frac{k_s}{2M} + \frac{k_y b_z^2 + b_y^2 k_s}{2I_x}$	(43)
T_y	Rotation about y-axis	$\omega_6 = \sqrt{\frac{k_s(b_x^2 + b_z^2)}{I_y}}$	(44)

8.3 Mount Deflections

If the object motions in all coordinates are as expressed in 8.1 (x, y, z, θ_x, θ_y, θ_z,) and if the coordinates of the mounting point (vibration isolators) are (X, Y, Z) in the equilibrium position, then their deflections (ΔX, ΔY, ΔZ) from equilibrium due to the applied force/torques are:

$$\begin{aligned} \Delta X &= x - \theta_z Y + \theta_y Z \\ \Delta Y &= y - \theta_x Z + \theta_z X \\ \Delta Z &= z - \theta_y X + \theta_x Y \end{aligned} \quad (45)$$

provided the deflections are small relative to the object dimensions.

However, if the effects of more than one disturbing force/torque are to be combined, the corresponding deflections of each mount must be combined *vectorially*, not be added algebraically, as in Equation (45).

General Comments

1. It is desirable to make sure that the disturbing forces and torques operate at frequencies sufficiently far removed from the computed natural frequencies, so that resonance conditions are avoided.
2. The compliance of the vibration mounts in compression and shear should be such that their combined compliance yields natural frequencies which are sufficiently lower than the frequencies of the disturbing forces and torques (hopefully at least by a factor of 2.5).

3. The displacements (max. deflections) of the mounts can be calculated from Equation (45) for any given single disturbing force or torque. If several force/torques act simultaneously, vector addition of forces in different directions is required, and Equation (45) cannot be used.
4. The case of a horizontal disturbing force has not been considered in this presentation.
5. Other things being equal, the best arrangement for the mounts is to arrange them so that their resultant force passes through the center of gravity of the equipment and that its line of action is a principal axis. If there is a resultant torque about the center of gravity, its direction should be about a principal axis through the center of gravity. However, if this arrangement is impractical, it need not be adhered to.

9.0 COMPLEX DRIVING FORCES

When the disturbing forces are neither sinusoidal nor suddenly applied, the vibration analysis becomes more complicated. While it is more difficult to give general guidelines or methods of analysis, one can consider every force-time variation as composed of components of different frequencies, each being a multiple of the basic (usually driving) frequency. Mathematically, this is known as expanding an arbitrary function into a Fourier series. Once these frequency components (harmonics) are determined, each one being sinusoidal at a different frequency, any component can be analyzed like a sinusoidal force. This can provide at least some understanding of the vibration phenomenon. Often the lowest-frequency (fundamental) component predominates and is the most important component to analyze. It is possible, however, that the design of the vibration isolation system will appear unfeasible on the basis of an analysis of only the fundamental component, whereas the exact analysis would show that a vibration isolation mounting can be useful; i.e., sometimes an analysis of components of several frequencies may be required [1]. This, however, may be quite difficult. In such cases, resolving an arbitrary force-time variation into several harmonics can provide some insight.

The following represents data in the Fourier series (decomposition into several harmonics) of some representative force-time variations in Figure 38, which are neither sinusoidal nor sudden. Each force is assumed to be a periodic function of the time;

$\lambda = \tau/T$, where τ is pulse width, T is the process period;
ω = fundamental frequency.

The Fourier expansions for these forcing functions are given in Table 2.

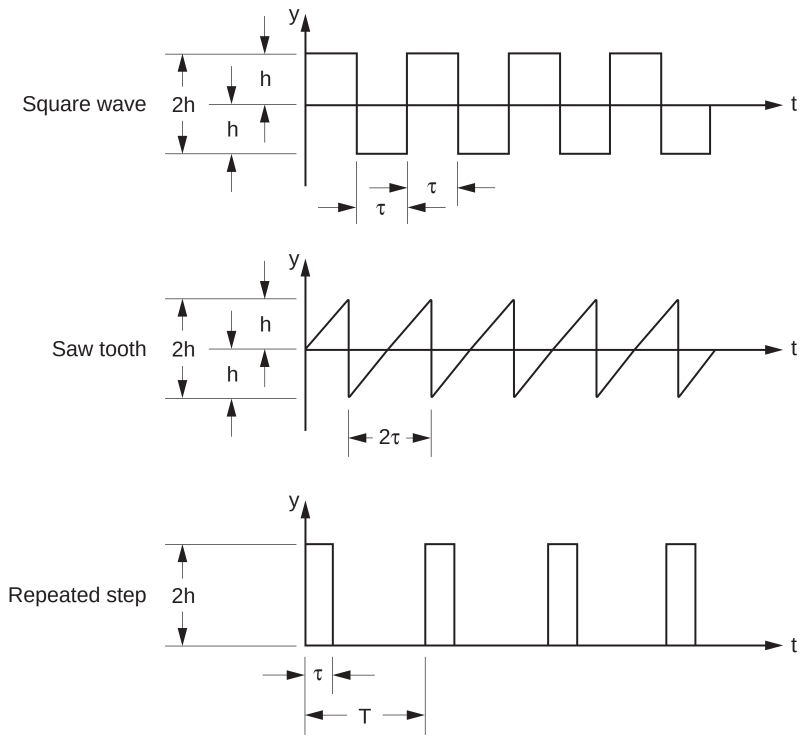

Figure 38 Typical Periodic Nonsinusoidal Vibratory Processes