The coupling concept can be illustrated on the example of a simpler "planar" system shown in Figure 28, which shows a mass supported by springs and constrained so that it can move only in the plane of the drawing [5]. Such a system has three coordinates which fully describe its configuration: translational coordinates x and y, and angular coordinate α. If the system is symmetrical about axis y, then when excited by a sinusoidal force F_y, in the vertical direction along the axis of symmetry, the object will behave as previously shown (Figure 1), namely by vibrating in the vertical (y) direction. However, if the force vector does not coincide with the axis of symmetry, then the vertical force would excite vibratory motions not only in the y-direction, but also in x and α directions. When the mass is excited by a horizontal force F_x, both horizontal ($\ddot{y}$) or longitudinal mode and pitching (α) vibratory motions are excited. These modes are said to be coupled when vibrations of one mode can be stimulated by a vibratory force or displacement in another. Coupling modes are in most cases undesirable. For example, many vibration-sensitive objects have the highest vibration sensitivity in a horizontal direction, while the floor vibrations are often more intense in the vertical direction. Coupling between the vertical and horizontal directions can be avoided by using vibration isolating mounts at each mounting point whose stiffness is proportional to the weight load acting on this mount (CNF mount) [1].

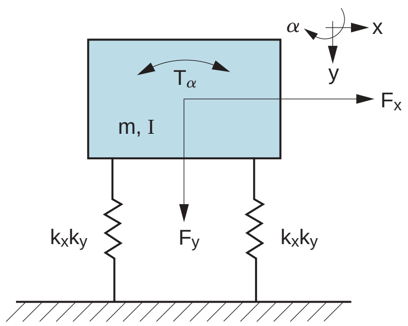

Figure 28 Planar (Three-Degrees-of-Freedom) Vibration Isolation System

T1 Technical Section

6.0 STATIC LOAD DISTRIBUTION CALCULATION

In order to calculate the weight distribution between the mounting points, the position of the CENTER OF GRAVITY (C.G.) has to be determined first. It is a simple task only for an axisymmetrical object. Position of the C.G. can be obtained by computation or experiment. The computational approach is feasible in most cases to the manufacturer who has all relevant drawings containing the data on mass distribution inside the object. The experiment is suggested by the definition of the C.G. as the point of support at which the body will be in equilibrium. For example, a small object can be supported on a peg; when in equilibrium, a vertical line drawn through the peg will pass through the C.G. Unfortunately, this method is applicable only to small objects. For large objects, such as machine tools, the object is mounted, for the C.G. location purposes, onto three load cells LC1, LC2, LC3, as shown is the plane view in Figure 29. If the weight loads as sensed by these load cells are W_1, W_2, W_3, respectively, then coordinates of the C.G. are as follows:

$$\left.\begin{aligned} x_{C.G.} &= \frac{x_1W_1 + x_2W_2 + x_3W_3}{W_1 + W_2 + W_3}\;; \\ y_{C.G.} &= \frac{y_1W_1 + y_2W_2 + y_3W_3}{W_1 + W_2 + W_3}\;. \end{aligned}\right\} \qquad (23)$$

After the C.G. position is known, weight distribution between the mounting points should be calculated. Such a calculation can be rigorously performed only for the case of an object with three mounting points (a statically-determinate problem). Unfortunately, only a relatively small percentage of objects requiring vibration isolation are designed with the "three point" mounting arrangement. If the number of the mounting points is greater than three, the accuracy of weight distribution calculations is suffering, unless the mounting surface of the floor is flat and horizontal and the mounting surface of the object is also flat. The tolerance on the "flatness" requirement should be a small fraction of the projected static deformations x_{st} of the selected vibration isolators.

For example, if the vertical natural frequency of the isolated object is f_n = 20 Hz, then, from Equation (4), x_{st} = 0.0625 cm or 0.625 mm.

Similarly, for f_n = 10 Hz, x_{st} = 2.5 mm, and
for f_n = 5 Hz, x_{st} = 10 mm.

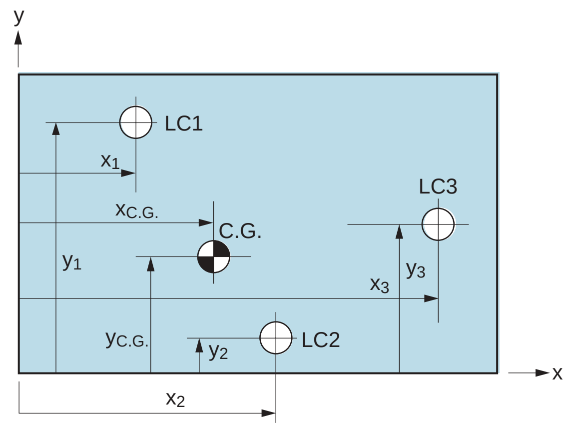

Figure 29 Setup for Experimental Finding of the C.G. Location

If the flatness tolerance is assumed to be 5% of x_{st}, then it can be realized only for small objects mounted on flat tables, especially for higher f_n.

With the assumption of perfectly flat mounting surfaces, load distribution between mounting points is given below for some basic object configurations.

In the case of a fully symmetrical four-point mounting system in Figure 30, the static load at each support point is equal to the total system weight (W) divided by the number of mounts (4). Thus,

$$P_1 = P_2 = P_3 = P_4 = \frac{W}{4} \tag{24}$$

Figure 31 describes a four-point mounting system where the C.G. is not at the geometric center. The static load for each of the four mounting points is:

$$\left.\begin{aligned} P_1 &= \left[\frac{BD}{AC}\right] W \qquad & P_3 &= \left[\frac{(C-D)\,B}{AC}\right] W \\ P_2 &= \left[\frac{(A-B)\,D}{AC}\right] W \qquad & P_4 &= \left[\frac{(A-B)\,(C-D)}{AC}\right] W \end{aligned}\right\} \tag{25}$$

Figure 32 illustrates a system which is symmetrical about one plane and the C.G. is offest from the geometric center on that plane. In such a system pairs of diagonals will carry equal loads. The static loading is given by the following equations:

$$\left.\begin{aligned} (P_1 + P_6) &= (P_4 + P_3) = \frac{W}{3} \\ (P_1 + P_2 + P_3) &= (P_4 + P_5 + P_6) = \frac{W}{2} \end{aligned}\right\} \tag{26a}$$

$$\left.\begin{aligned} \text{where: } P_1 = P_4 &= \left[\frac{(2C - 3A + D)}{6C}\right] W \\ P_3 = P_6 &= \left[\frac{3A - D}{6C}\right] W \\ P_2 + P_5 &= \frac{W}{3} \\ \text{and} \quad P_2 = P_5 &= \frac{W}{6} \end{aligned}\right\} \tag{26b}$$

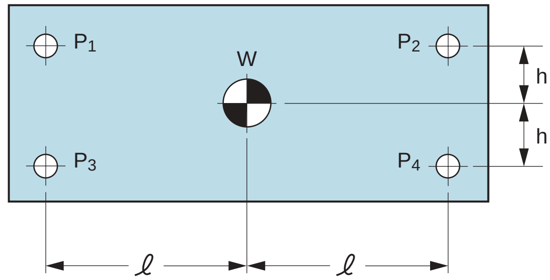

Figure 30 Symmetrical Four-Point Mounting System

NOTE: It is assumed for the purpose of Figures 30, 31 and 32, that the loaded surface as supported by the points P_1, P_2, P_3, P_4, P_5 and P_6, is in the horizontal plane.

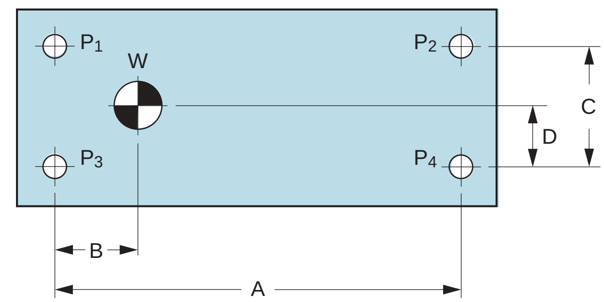

Figure 31 Nonsymmetric Four-Point Mounting System

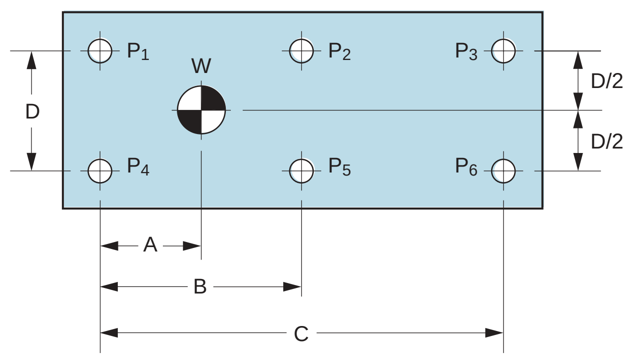

Figure 32 Nonsymmetric Six-Point Mounting System

Unfortunately, schematics of Figures 30-32 are not often representative for mounting of real (especially big and heavy) objects, the floors are far from perfectly flat, and the weight load distribution is usually determined by "guesstimation", very crudely. In many objects, such as machine tools, etc., there are heavy moving parts (e.g., tables) whose repositioning is changing the weight distribution. Since even in a case when the load distribution is precisely known the constant stiffness vibration isolators with the required stiffness values are not available, and since the actual stiffness can be unpredictably ±17% different from the nominal stiffness, the stated above (in Section 5) decoupling condition is very infrequently satisfied, thus requiring use of softer isolators or resulting in an inferior isolation.

6.1 Advantages of CNF Vibration Isolators

Constant Natural Frequency (CNF) vibration isolators have their stiffness proportional to the weight load acting on them, see 4.0 above. Obviously, there is no need in determining position of the C.G., in maintaining perfectly flat floors, or in computing or guesstimating the weight distribution between the mounting points of the isolated object. Identical CNF mounts would automatically satisfy the decoupling condition thus resulting in a better performance of the vibration isolation system while using stiffer isolators. Figure 33 [1] shows amplitudes of relative vibratory motion between the grinding wheel and the workpiece of a precision surface grinder while the floor is vibrating in vertical direction with amplitude of 5 μm in a frequency range of 10-35

Hz. When the machine is installed on five linear isolators with rubber flexible elements selected in accordance with the manufacturer's recommendations, different for different mounting points (line 2, f_n = 15 Hz), the maximum amplitude of the relative vibrations (resulting in waviness of the ground surface) was 0.35 μm. However, when the grinder was installed on five indentical CNF isolators with rubber flexible elements (line 1, f_n = 20 Hz, or about two times stiffer than the linear isolators), the maximum relative vibration amplitudes was 0.25 μm, about 30% lower.

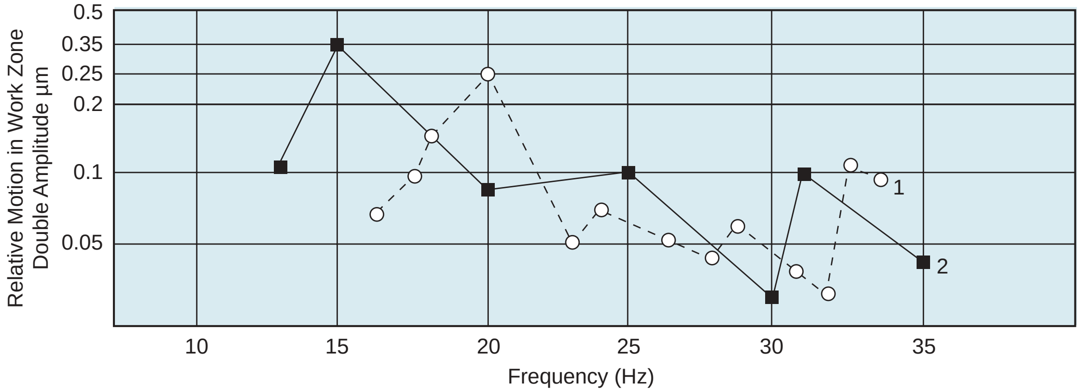

Figure 33 Amplitude of Relative Motion in Work Zone with: 1 - Regular (Linear) Isolators; 2 - CNF Isolators

7.0 CONNECTIONS OF SPRING ELEMENTS

7.1 Springs in Parallel

These combine like electrical resistance in series. This is the case when several springs support a single load, as shown in Figure 34. The springs are equivalent to a single spring, the spring constant of which is equal to the sum of the spring constants of the constituent springs. The spring constant k of the single equivalent spring is given by:

$$k = k_1 + k_1 + k_1. \qquad (27)$$

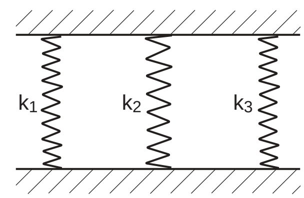

Figure 34 Parallel Connection of Springs

7.2 Springs in Series

The series connected springs in Figure 35 combine like electrical resistances in parallel. The equivalent single spring is softer than any of the component springs. The spring constant k of the equivalent single spring is given by:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}\,. \qquad (28)$$

If n springs are in series, this formula is readily extended to:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots\ldots + \frac{1}{k_n}\,. \qquad (29)$$

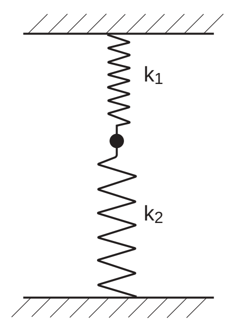

Figure 35 Series Connection of Springs

7.3 Spring Connected Partly in Parallel and Partly in Series

Obtain equivalent spring constants for each set of parallel or series springs separately and then combine. For example, in Figure 36, the springs k_1 and k_2 are equivalent to a single spring, the spring constant of which, k_{e1}, is given by:

$$\frac{1}{k_{e1}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1 + k_2}{k_1 k_2} \quad \text{or} \quad k_{e1} = \frac{k_1 k_2}{k_1 + k_2} \qquad (30a)$$

The three springs, k_3, k_4, k_5 in parallel, are equivalent to a single spring, the spring constant of which, k_{e2}, is given by:

$$k_{e2} = k_3 + k_4 + k_5 \qquad (30b)$$

Now equivalent springs k_{e1} and k_{e2} are in series. Hence, the spring constant k of the equivalent spring for the entire system is:

$$\frac{1}{k} = \frac{1}{k_{e1}} + \frac{1}{k_{e2}} \quad \text{or} \quad k = \frac{(k_1 k_2)(k_3 + k_4 + k_5)}{k_1 k_2 + (k_1 + k_2)(k_3 + k_4 + k_5)} \qquad (30c)$$

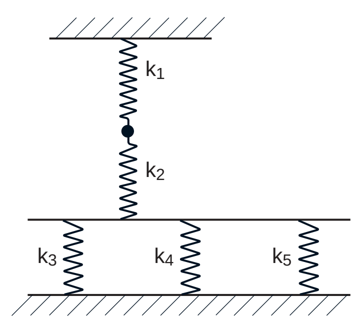

Figure 36 Mixed Connection of Springs