$$A = \frac{F_o}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}} = \frac{F_o / k}{\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} \qquad (6)$$

RESONANCE — It is seen in Figure 7 that displacement and stress levels tend to build up greatly when the forcing frequency coincides with the natural frequency, the build-up being restrained only by damping. This condition is known as *RESONANCE.*

In many cases, the forced vibration is caused by an unbalanced rotating mass, such as the rotor of an electrical motor. The degree of unbalance can be expressed as distance e between the C.G. of the rotor and its axis of rotation. The vertical component of the centrifugal force generated by the unbalanced rotor (mass M) is

$$F_{c.f.} = M\omega^2 e \sin \omega t = 4\pi^2 M f^2 e \sin 2\pi t, \quad (7)$$

where ω is angular speed of rotation in rad/sec and f is the number of revolutions per second. In case of vibration excitation by the unbalanced rotor, combining of (6) and (7) results in

$$A = \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}}$$

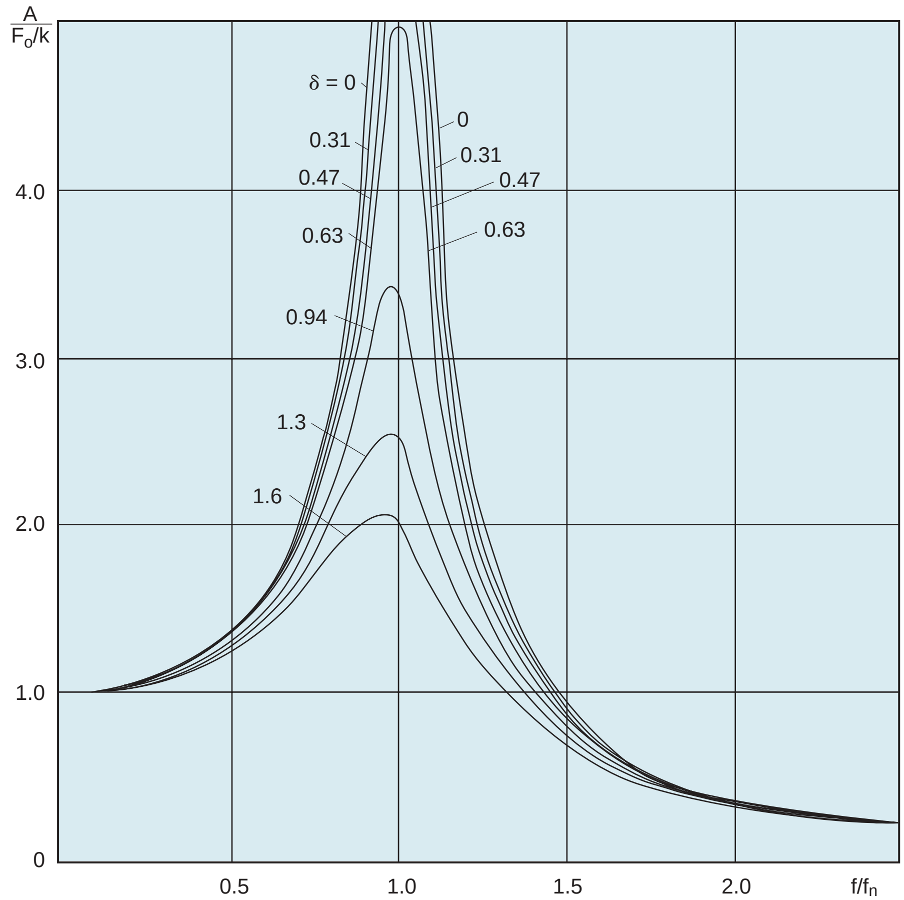


Figure 7 Amplitude-Frequency Characteristics of Massive Block Motion in Figure 6

$$= \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} = \frac{Me}{m}\,\frac{f^2 / f_n^2}{\sqrt{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}}, \qquad (6a)$$

where m is the total mass of the object. Expression (6a) is plotted in Figure 8 for several values of damping (δ).

3.0 Vibration Isolation

Although *VIBRATION ISOLATION* is a very large area of vibration control, there are two most widely used techniques of vibration isolation:

– Reduction of transmission of vibratory or shock forces from the object, in which these forces are generated, to the base; and

– Reduction of transmission of vibratory motions of the base to the work area of vibration-sensitive objects.

These techniques are similar, but also quite different. They both deal with *TRANSMISSIBILITY* or *TRANSMISSION RATIO*. There are several transmission ratios. Usually these refer to the ratios of the maximum values of the transmitted force or displacement to the maximum values of the applied force or the forced motion. The important direction of transmission is from the object to the base for the force isolation, or from the base to the object for the motion isolation.

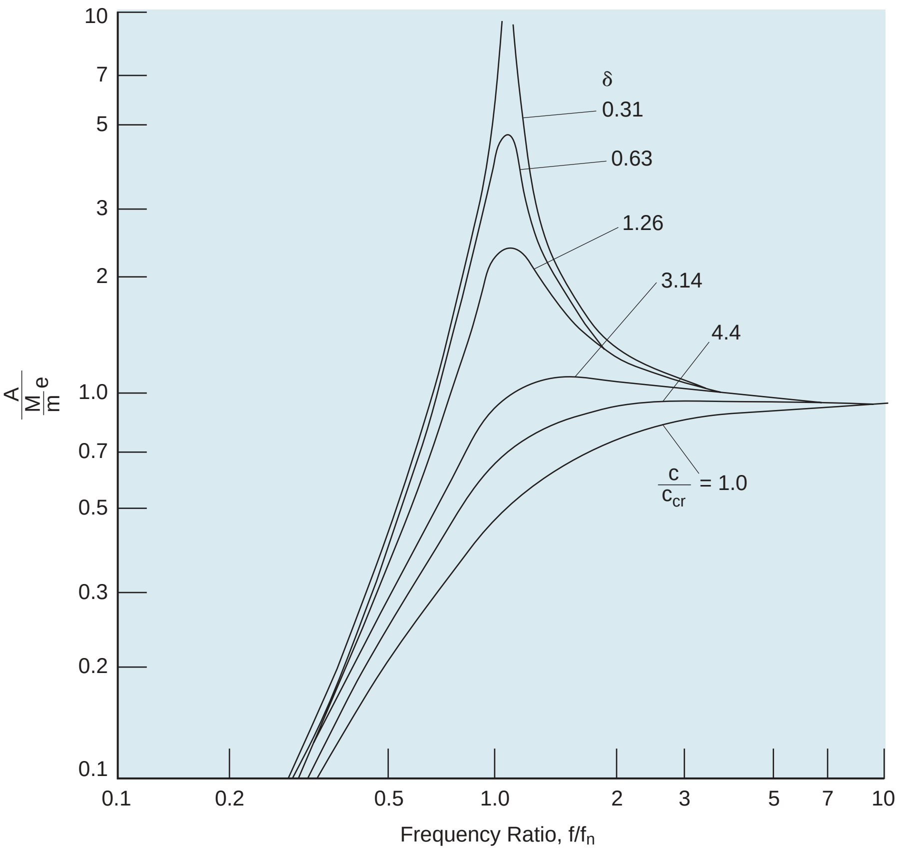


Figure 8 Amplitude-Frequency Characteristics of Massive Block Motion in Figure 6 Excited by an Unbalanced Rotor

3.1 Vibration Isolation of Vibration-Producing Products

Figure 9 shows a simplified single-degree-of-freedom model of a vibration isolation system. While in models in Figure 1 and Figure 2, the base (foundation) is shown as having infinite mass, in Figure 9 model the foundation has a finite mass m_f. If the force $F(t) = F_o \sin 2\pi ft$ is generated in the object (mass m), the force transmissibility μ_F from the object to the foundation is equal to the motion transmissibility μ_x from the foundation to the object and is expressed as

$$\mu_F = \mu_x = \frac{|F_f|}{|F_o|} = \frac{|x_1|}{|x_2|} = \frac{m_f}{m + m_f}\sqrt{\frac{1 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}}\ . \qquad (8)$$

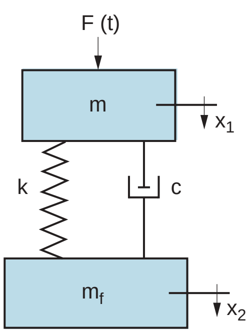


Figure 9 Dynamic Model of a Basic Vibration Isolation System

This expression (for $m_f = \infty$) is plotted in Figure 10 which shows that "isolation" of the force source or the condition of $\mu_F < 1$ develops at frequencies greater than $f = 1.41f_n$ and fast improving with further increasing of the frequency ratio f/f_n. The maximum transmissibility occurs at the resonance when the frequency ratio $f/f_n = 1$. At resonance ($f = f_n$), the transmissibility at not very high damping is expressed as

$$(\mu_F)_{max} = (\mu_x)_{max} \approx \frac{m_f}{m + m_f}\,\frac{\pi}{\delta}\ . \qquad (9)$$

While increasing of damping is beneficial at and around the resonance, the isolation at high frequencies deteriorates with increasing damping δ. This effect must be considered in designing the isolation system for a given application. Still, a reasonable increase of damping is important since it makes the system more robust if subjected to inevitable spurious excitations. Also, the higher damping improves behavior of the system if the object generates forces in a broad frequency range; e.g., as unbalanced motor(s) generating continuously changing excitation frequency during its acceleration phase. It should be considered that the transmissibility curves in Figure 10 are plotted for viscous damping in the isolators. Damping in elastomeric and wire-mesh (or cable) elements is different, so-called hysteretic damping. This latter type of damping does not affect the preresonance and the resonance behavior of the system, but demonstrate only a minimum deterioration of the isolation at high frequencies even for highly-damped isolators (more in [1]).

As mentioned before, the goal of vibration isolation of vibration-sensitive objects from the base vibration is to reduce relative vibratory displacements in the work zone. Transmissibility of the base motion into the relative vibrations $\theta = x_1 - x_2$ is (for any value of m_f):

$$\mu_{rel} = \frac{|\theta|}{|x_2|} = \frac{\dfrac{f^2}{f_n^2}}{\sqrt{\left(1 - \dfrac{f^2}{f_n^2}\right)^2 + \left(\dfrac{\delta}{\pi}\,\dfrac{f}{f_n}\right)^2}}. \qquad (10)$$

Expression (10) is plotted in Figure 11. It is clear that transmissibility of low frequency (as compared with the natural frequency) foundation vibrations into the relative vibrations is very small (since at low frequencies the motions are very slow and the object is moving following the vibrating foundation).

ISOLATION EFFICIENCY — Isolation is the percent of vibration force that is not transmitted through the vibration mounts and which improves with increasing frequency ratio. Isolation efficiency of 81.1% corresponding to a frequency ratio of 2.5, is generally adequate as shown in Table 2. Figure 12, the basic vibration chart, gives static deflection vs. frequency and % of vibration isolation ($1 - \mu_F$). It is useful for selection of vibration isolators/mounts and for calculations (see Section 11).

A more complete treatment of this case of vibration isolation, considering more complex and more realistic (several degrees of freedom) models is given in [1].

Table 2: VIBRATION ABSORPTION

Frequency Ratio	Vibration Absorption, Percent	Results Attained
10.0	98.9	excellent
4.0	93.3	excellent
3.0	87.5	very good
2.5	81.1	good
2.0	66.7	fair
1.5	20.0	poor
1.4	0	none
1.0	(resonance)	worse than with no mountings

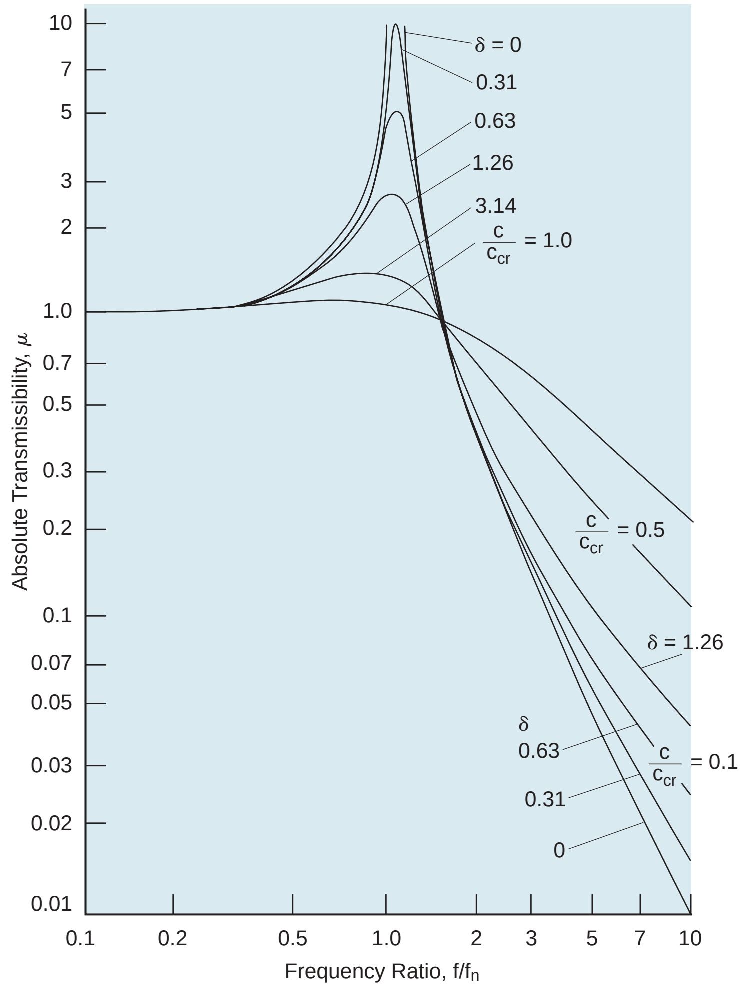


Figure 10 Force/Motion Transmissibility in Figure 9 System

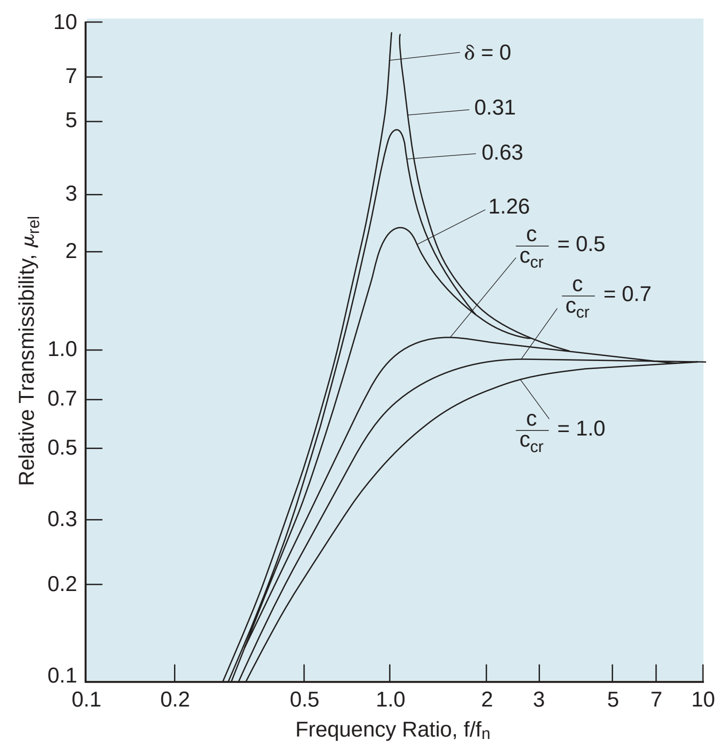


Figure 11 Transmissibility of Vibratory Base Motion to Relative Vibratory Motion in the Work Zone

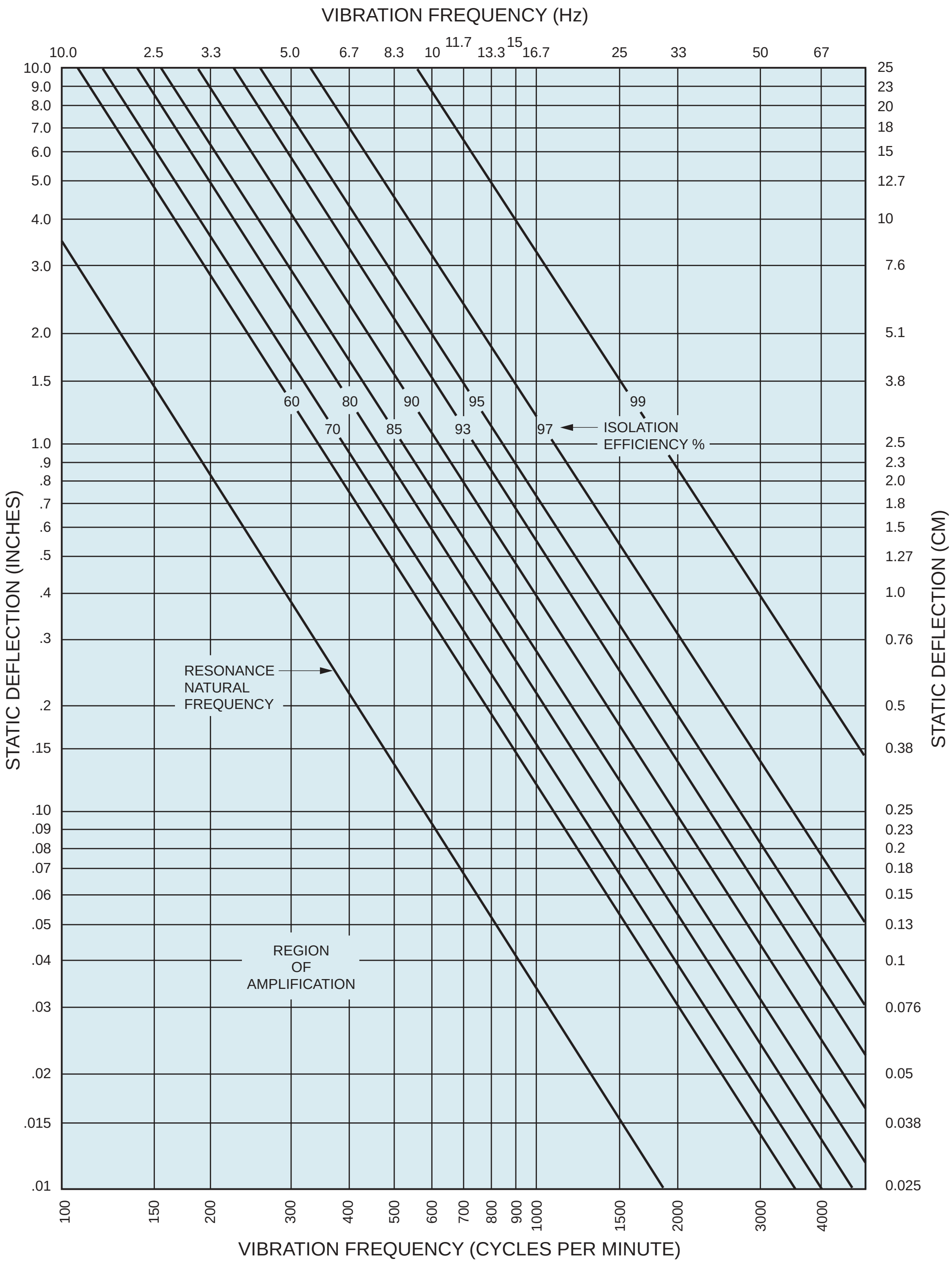


Figure 12 Vibration Frequency vs Static Deflection of Isolators vs Isolation Efficiency

3.2 Vibration Isolation of Vibration-Sensitive Objects

Since, for this group of objects, the relative vibrations in the work zone are determined by dynamic characteristics of the object itself, a model in Figure 13 should be considered. Floor (foundation) vibration $x_1 = x_{10} \sin 2\pi ft$ is transmitted through vibration isolators (stiffness k_v, damping coefficient c_v) to frame/bed of the object (mass M_B) causing its vibrations $x_B = x_{B0} \sin 2\pi ft$. The work zone of the object is between the frame/bed and its "upper unit", mass M_u (e.g., tool head of a machine tool or illumination unit of a photo-lithography tool). Stiffness k_m and damping coefficient c_m describe structural dynamic characteristics of the object, whose structural natural frequency is

$$f_m = \frac{1}{2\pi} \sqrt{\frac{k_m (M_u + M_B)}{M_u M_B}} \; . \qquad (11)$$

Accordingly, transmissibility of the vibratory motion of the foundation into the work zone can be expressed as a product of (transmissibility μ_x of the foundation motion X_1 to the frame motion from expression (7) where $x_2 = X_1$, $x_1 = X_B$; and $m = M_B$) and (transmissibility of the frame motion X_B to the relative motion X_{rel} in the work zone μ_{rel} from expression (8) where $x_2 = X_B$, $\theta = X_{rel}$, and $f_n = f_m$ from expression (11)). This operation is illustrated in Figure 14.

In Figure 14, the plot (a) is maximum intensity a_o of floor vibration (displacements amplitudes compounded from numerous on-site measurements). It is shown in [1] that for a majority of manufacturing plants $a_o \approx 2.5 \mu m$ in the 4-30 Hz range and is much smaller outside of this range for vertical floor vibrations, and $a_o \approx 2.0\ \mu m$ in the 4-20 Hz range and much smaller outside of this range for horizontal floor vibrations. For high precision facilities, the levels of allowable floor vibrations are recommended by BBN plots in Figure 15. The next plot (b) in Figure 14 illustrates transmissibility from the floor to the object frame for three cases: a - the object installed on rigid mounts (e.g., jack mounts or rigid isolator mounts); b - the object installed on softer, isolating mounts (lower f_n) with the same degree of damping (height of the resonance peak) as the mounts in a; c - the same f_n as in b, but greater damping. The third plot (c) illustrates transmissibility from the frame of the object into its work zone; f_m is the structural natural frequency of the object. The bottom plot shows the product of the previous three plots. An installation is considered successful if the vibration amplitude in the work zone does not exceed the allowable amplitude Δ_o.

It can be seen that a rigid installation results in two peaks of the relative vibration amplitude, which often exceed the tolerance. Both peaks are reduced by using soft isolator mounts: the second one due to reduced transmissibility at high frequencies per expression (6), and the first one due to lower sensitivity of the object structure to lower resonance frequency of the object on softer isolating mounts. It is clear, that increasing damping also results in reduced relative vibrations. Accordingly, the requirement for an adequate vibration isolation of a vibration-sensitive object is formulated not as a required upper limit of the natural frequency f_n, but as a required upper limit of the "Isolation Criterion" Φ,

$$\Phi = \frac{f_n}{\sqrt{\delta}} . \tag{12a}$$

The magnitude of this criterion can be calculated if vibration sensitivity of the object in the frequency range of interest is measured and its tolerance is assigned, see [1]. The object is properly isolated if

$$\Phi < \sqrt{\frac{\Delta_o f^2}{\pi X_f \mu_f}} , \tag{12b}$$

where Δ_o is the maximum tolerated vibratory displacement in the work zone of the object, X_f is the maximum amplitude of floor vibration with frequency f; μ_f is the transmissibility into the work zone at frequency f (ratio of relative vibration amplitude in the work zone to amplitude of the object frame vibration at frequency f). According to this criterion widely validated by practical applications, stiffness of isolators for a given installation can be increased (usually, a very desirable feature) if the isolators have higher damping.

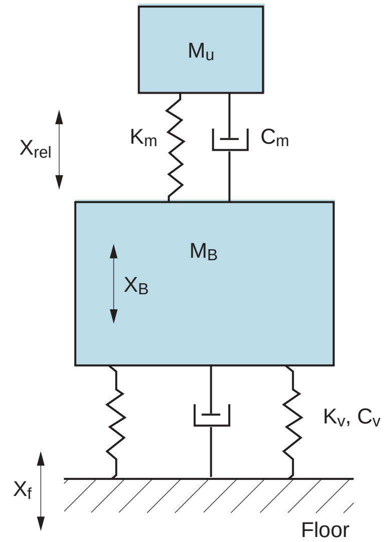
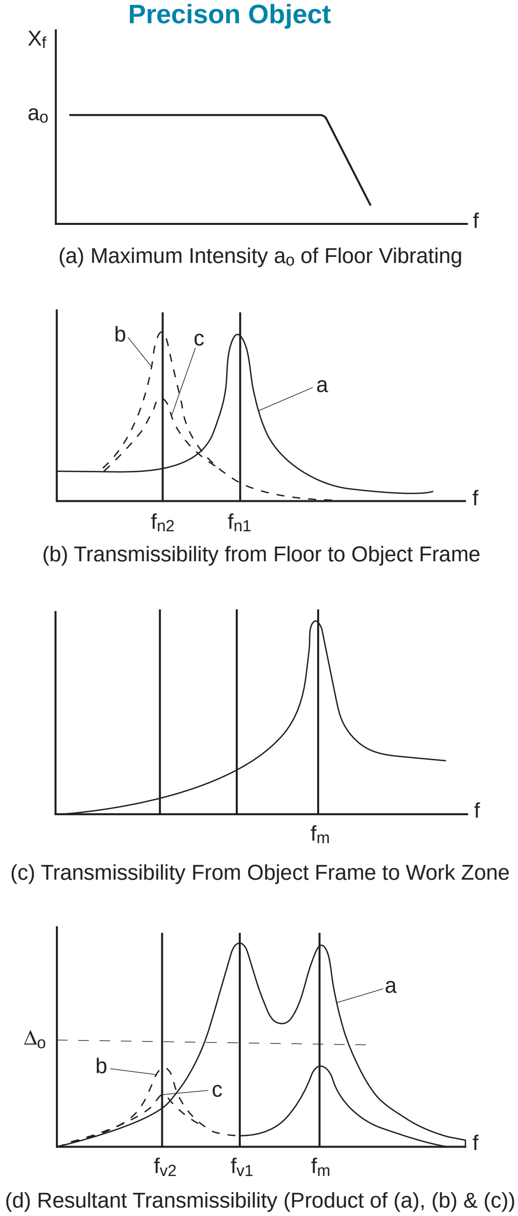


Figure 13 Two-Mass Dynamic Model for Vibration Sensitivity of Precison Object



(a) Maximum Intensity a_o of Floor Vibrating

(b) Transmissibility from Floor to Object Frame

(c) Transmissibility From Object Frame to Work Zone

(d) Resultant Transmissibility (Product of (a), (b) & (c))

Figure 14 Model of Vibration Transmission from Floor to Work Zone

Thus, while vibration isolation of the force-producing objects requires reducing natural frequency in accordance with nomogram in Figure 12, isolation of a vibration-sensitive object can be successful even when some part of the system is at resonance, provided that the natural frequency of the isolation system and its damping are properly selected. Vibration isolation in the latter case is greatly simplified if structural stiffness and structural natural frequency of the vibration-sensitive object are enhanced.

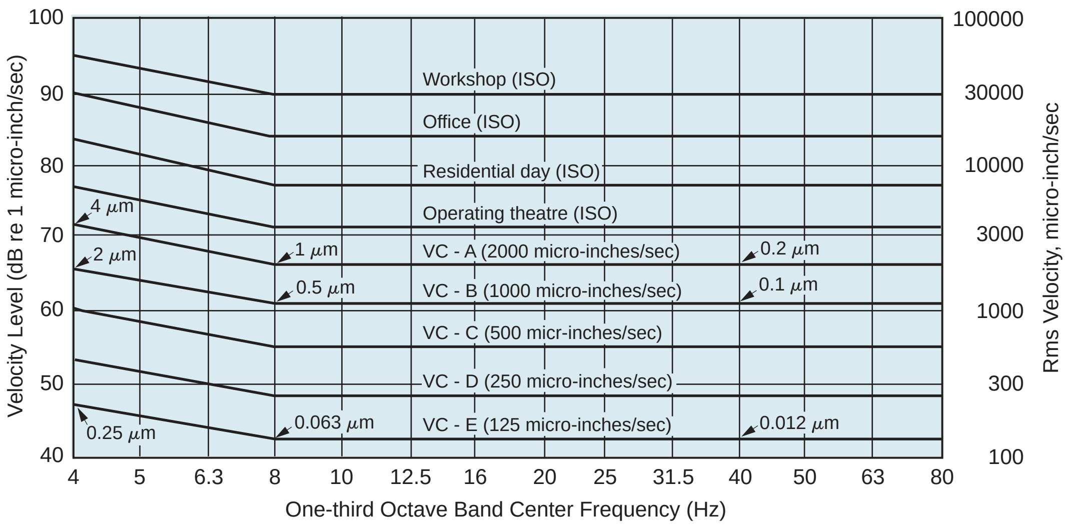


Figure 15 BBN Vibration Criteria (VC) for Installation of Precision Equipment

3.3 Shock Isolation

The information in this section has been taken from [2] with permission of the publisher.

It is often necessary to determine the effectiveness of a shock isolator as well as the magnitude of the acceleration experienced by elements of the protected equipment. Figure 16, similar to Figure 13, describes the system experiencing a velocity shock as illustrated by the displacement-time curves of Figure 17.

The displacement of equipment (y) supported by isolators and subjected to a velocity shock (V) is expressed by the following equation:

$$y = V\left(1 - \frac{1}{2\pi f_y}\sin 2\pi f_y t\right) \tag{13}$$

where $f_y = \frac{1}{2\pi}\sqrt{\frac{k_y}{m_y}}$ is the natural frequency, Hz, of the elastic system consisting of chassis (m_y) and isolator (k_y). Double differentiation of equation (13) yields the acceleration experienced by the equipment chassis during shock. This is designated the transmitted acceleration and is expressed as:

$$\ddot{y}_0 = 2\pi f_y V \tag{14}$$

The units of acceleration $\ddot{y}_0$, are linear distance (inches, m, etc) per second per second. This equation can be expressed another way, using more convenient engineering units, as:

$$\text{Transmitted Shock} = \frac{\ddot{y}_0}{g} = \frac{2\pi f_y V}{386} = \frac{f_y V}{61.4}, \tag{15}$$

where: V = shock velocity change, in/sec.
f_y = natural frequency of isolator, Hz.
$\ddot{y}_0/g$ = maximum acceleration experienced by chassis, expressed as a dimensionless multiple of the acceleration due to gravity.

Thus, the maximum acceleration of the chassis during shock, is directly proportional to the magnitude of the velocity change and to the natural frequency of the isolator. Figure 18 is a graphic representation of the maximum transmitted acceleration computed from Equation (15).

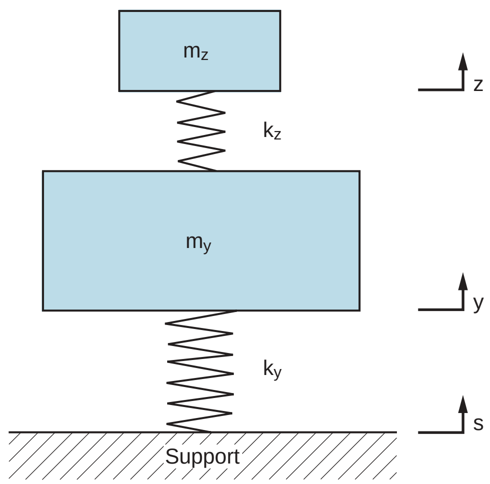


Figure 16 Schematic Representation of Equipment, Comprised of Chassis my and Element $m_z k_z$, Mounted Upon Isolator k_y.

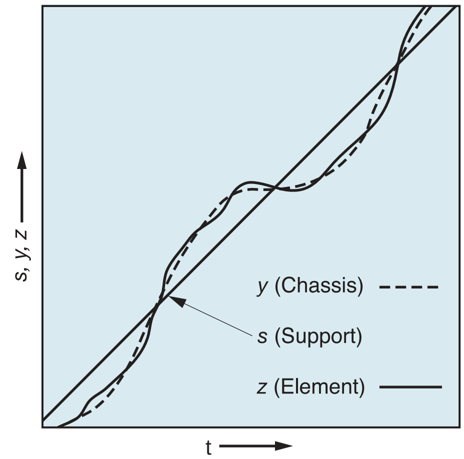


Figure 17 Displacement-Time Curves for Support, Chassis, and Element of Equipment (Inelastic Impact)

The maximum acceleration experienced by the chassis of the mounted equipment, as indicated in Figure 18, should not be confused with the maximum acceleration experienced by various elements of the equipment. The latter is equal to the product of the maximum chassis acceleration y_0 and the amplification factor A_0, which is defined as the ratio of the maximum acceleration of the element ($\ddot{z}_0$) to the maximum acceleration of the chassis ($\ddot{y}_0$) and is given by:

$$A_0 = \frac{\ddot{z}_0}{\ddot{y}_0} \qquad (16)$$

In the absence of damping, A_0 is a function only of the element's natural frequency (f_z) and the isolator's natural frequency (f_y). For an undamped system, shock transmissibility (T_s) is related to the amplification factor (A_0) as follows:

$$T_s = A_0 \left(\frac{f_y}{f_z}\right) \qquad (17)$$

where shock transmissibility (T_s) is the ratio of the maximum acceleration of the mass element, m_z, to the maximum acceleration of the same element which would occur if the isolator's spring constant, k_y, were infinitely rigid.

Using values for the amplification factor A_0 as determined in [3], and plotted for a range of values of damping ratio, shock transmissibility can be determined for a damped system as shown in Figure 19. The damping between m_z and m_y is assumed to be constant at one percent critical damping (δ = 0.063). However, wide variations in the degree of damping have little effect on the results. Figure 20 gives the amplification factor A_0 for the system shown in Figure 16 when the support experiences velocity shock as illustrated in Figure 17. The factor A_0 is the ratio of the maximum acceleration of mass m_z to the maximum acceleration of mass m_y.

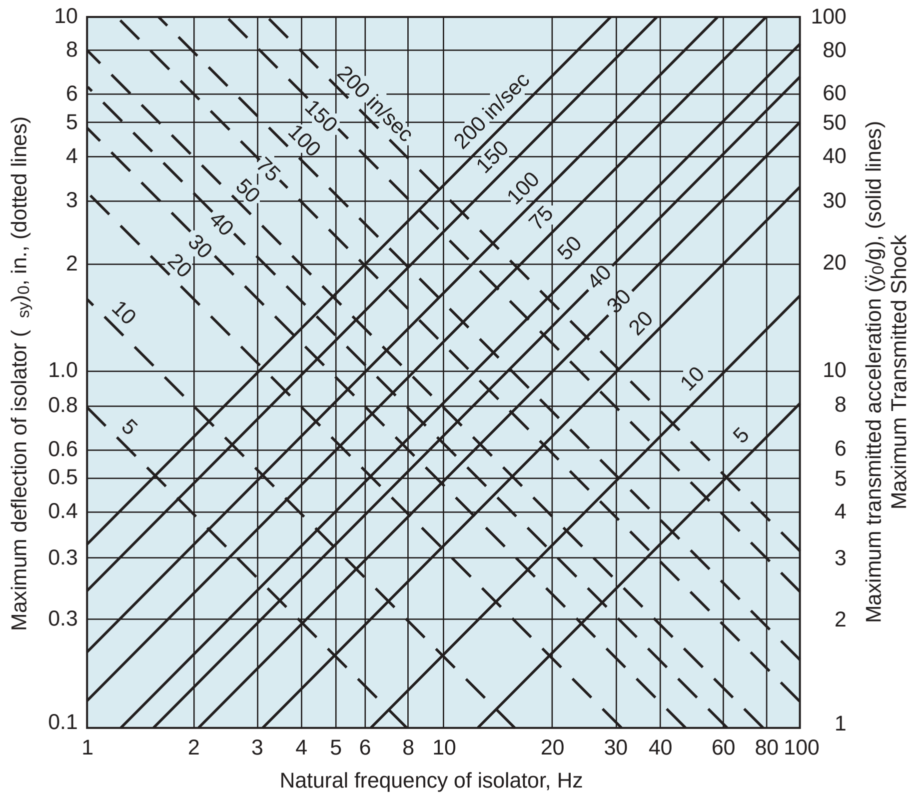


Figure 18 Maximum Acceleration of Chassis m_y and Maximum Deflection of Linear Isolator k_y Shown in Figure 16, When Support Experiences Velocity Shock as Illustrated in Figure 17.

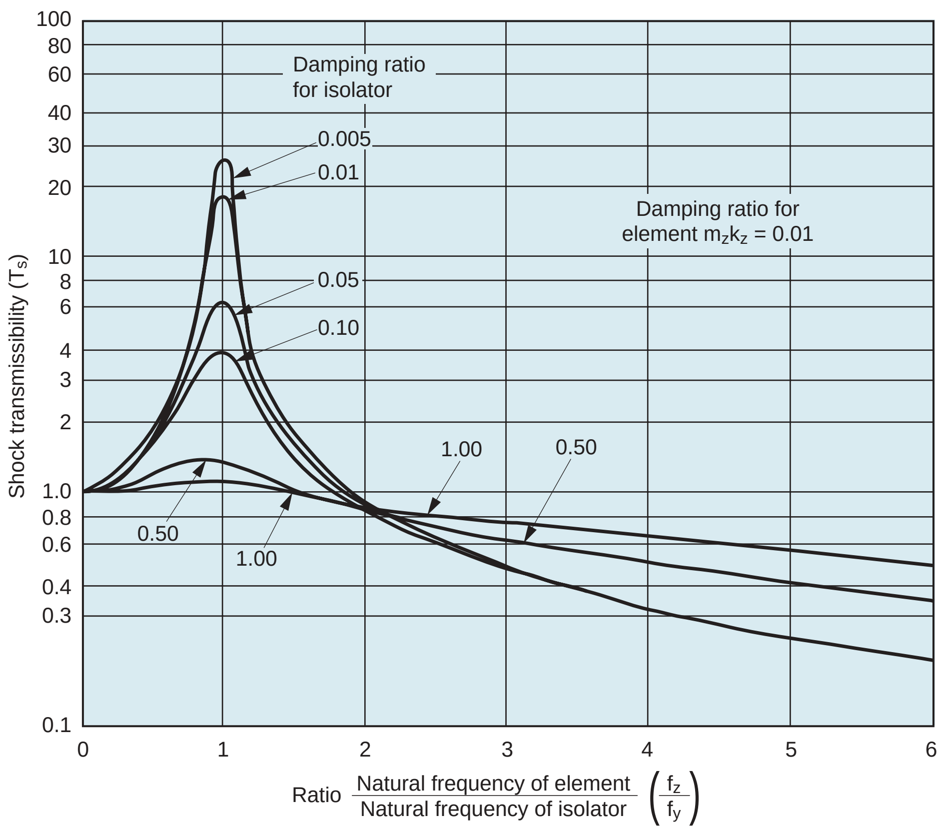


Figure 19 Shock Transmissibility for System Shown in Figure 13, When Subjected to Velocity Shock as illustrated in Figure 17 [3].

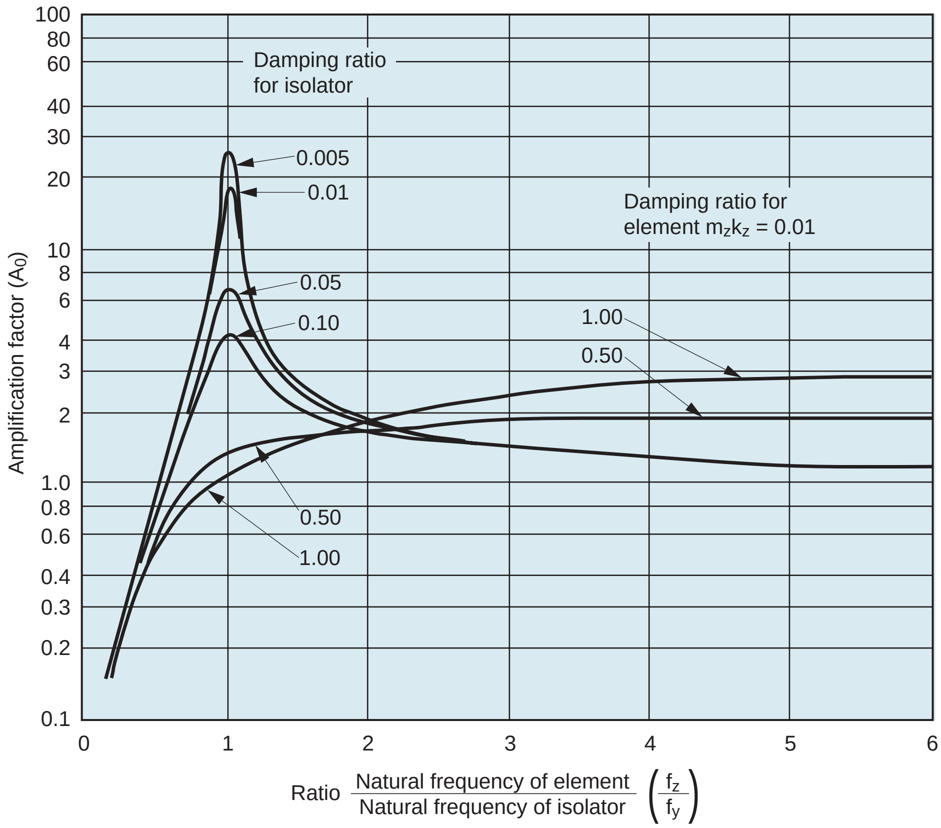
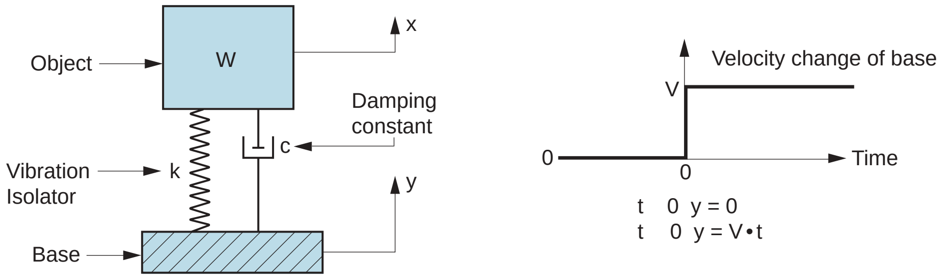


Figure 20 Amplification Factor for System Shown in Figure 16 When Subjected to Velocity Shock as Illustrated in Figure 17

3.3.1 Shock Motion of Base (Base Suddenly Stops or Accelerates)

The time history of the sudden acceleration process of the base in Figure. 21(a) is shown in Figure 21(b). The analytical results taken from [3] are also applicable to the object (equipment unit) dropping from a height onto a hard surface.



(a) System **(b) Motion of Base**

Figure 21 Vibration Isolation System for Object W (a) Subjected to Shock Motion of Base with Time History (b)

If: V = sudden velocity change of base, in/sec or m/sec
c/c_{cr} = $\delta/2\pi$ = damping ratio where δ is log decrement
f_n = undamped natural frequency of system, Hz
g = gravitational constant, 386 in/sec^2 = 9.81 m/sec^2
d_{max} = max. isolator deflection, measured from equilibrium position, in. or m
d_{st} = static isolator deflection = W/k, in. or m
a_{max} = maximum acceleration of object, in/sec^2 or m/sec^2

then, for $0 \leqslant c/c_{cr} \leqslant 0.2$ or $0 \leqslant \delta \leqslant 1.25$,

$$\frac{d_{max}}{d_{st}} = \frac{a_{max}}{g} = \frac{2\pi f_n(1 - c/c_{cr})}{g} \tag{18}$$

Figure 22 illustrates Equation (18). When the damping is small, maximum force transmitted to equipment is very nearly kd_{max}.

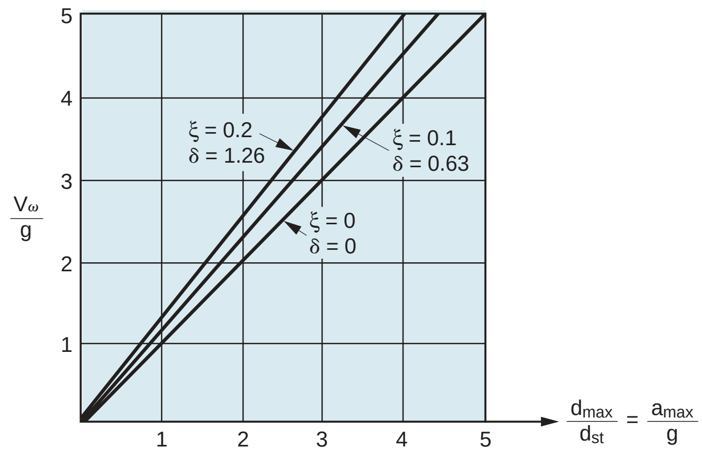


Figure 22 Shock Effect at Different Damping Values

3.3.2 Sudden Impact on Equipment [3]

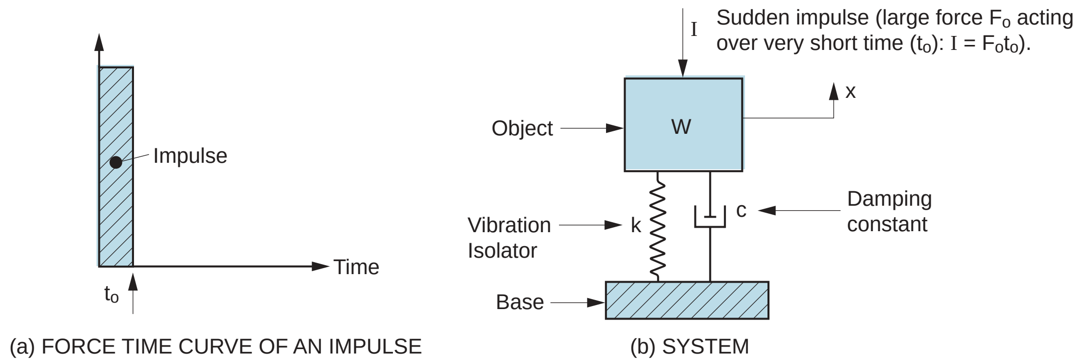


Figure 23 Vibration Isolation System of Object W (b) Subjected to Sudden Impact on the Object with Time History (a)

Sudden impact, or a sharp blow is characterized by a large force (F_o) acting for a short period of time (t_o) as shown in Figure 23(a). For practical purposes, suddenness is taken to mean that t_o is small in comparison with the natural period of vibration of the system in Figure 23(b). The *impulse*, I, is defined as the area under the force-time curve; i.e.,

$$I = F_o t_o \text{ lb-sec or kg m/sec} \tag{19}$$

Application of impulse I results in a sudden downward velocity V of the object,

$$V = Ig/W. \tag{20}$$

The maximum isolator deflection and the maximum acceleration of the object can be obtained by substituting V into Equation (18).

4.0 NONLINEARITIES

The equations previously given for transmissibilty (Section 3.1) make certain assumptions which may not always be valid. For example, it is assumed that the damping is viscous or *linear* (resistance to relative motion is proportional to the relative velocity). The assumption greatly simplifies the analysis. However, the damping provided by wire mesh is a combination of localized frictional losses by individual wires and hysteresis in the cushion itself. Damping in elastomeric materials has similar characteristics. In practical terms, this means that the damping is a function of displacement in addition to velocity, and the terms describing the damping in the equations of motion are *nonlinear*. At resonance, where the displacement is large, the damping is high. In the isolation band, where displacement is small, the damping is negligible. This condition gives the best of both worlds as damping is only desirable under resonance conditions. Thus, the idealized curves in Figure 10 are on the conservative side since they show deterioration of isolation in the high frequency (after resonance) range.