1.5 Principles of Noise Reduction

A good vibration isolation system is reducing vibration transmission through structures and thus, radiation of these vibration into air, thereby reducing noise.

There are many ways to reduce noise. One of the most practical and effective may be the use of vibration mounts. As a general rule, a well-designed vibration isolator will also help reduce noise. In the case of panel flutter, for example, a well-designed vibration mount could reduce or eliminate the noise. This can be achieved by eliminating the flutter of the panel itself, or by preventing its transmission to ground, or by a combination of the two. The range of audible frequencies is so high that the natural frequencies of a vibration mount can usually be designed to be well below the noise-producing frequency.

In order to reduce noise, try to identify its sources; e.g., transformer hum, panel flutter, gear tooth engagement, rotor unbalance, etc. Next, identify the noise frequencies. Vibration isolators for these sources designed in accordance with the guidelines for vibration and shock control may then act as barriers either in not conducting the sound, or in attenuating the vibration which is the source of the noise.

2.0 BASIC DEFINITIONS AND CONCEPTS IN VIBRATION AND SHOCK ANALYSIS

2.1 Kinematic Characteristics

COORDINATE — A quantity, such as a length or an angle, which defines the position of a moving part. In Figure 1, x is a coordinate, which defines the position of the weight, W.

DISPLACEMENT — A change in position. It is a vector measured relative to a specified position, or frame of reference. The change in x (Figure 1) measured upward, say, from the bottom position, is a displacement. A displacement can be positive or negative, depending on the sign convention, translational or rotational. For example, an upward displacement may be positive, and a downward displacement negative. Similarly, a clockwise rotation may be positive and a counterclockwise rotation negative. Units: inches, feet, meters (m), millimeters (mm), or, in the case of rotations: degrees, radians, etc.

VELOCITY — The rate of change of displacement. Units: in/sec, mph., m/sec, etc. Velocity is a vector whose magnitude is the *SPEED*. Angular velocity might be measured in radians/sec or deg/sec, clockwise or counterclockwise.

ACCELERATION — The rate of change of velocity. Units: in/sec^2, m/sec^2, etc. It is a vector and has a magnitude and direction. Angular acceleration might be measured in rad/sec^2 or deg/sec^2, clockwise or counterclockwise.

VIBRATORY MOTION — An oscillating motion; such as, that of the weight W, in Figure 1.

SIMPLE VIBRATORY MOTION — A form of vibratory motion, which as a function of the time is of the form $x = a \sin \omega t$, where a and ω are constants. The maximum displacement, a, from the mean position (x = 0) is the *AMPLITUDE*; the *FREQUENCY* (rate at which the motion repeats itself) is $f = \omega/2\pi$ cycles/sec, where *ANGULAR FREQUENCY* ω has the dimensions of rad/sec, and frequency f has the dimensions of reciprocal time; e.g. reciprocal seconds 1/sec. Such motion is also called harmonic or sinusoidal motion.

PERIOD, CYCLE — The interval of time within which the motion repeats itself. In Figure 5, this is T seconds. The term cycle tends to refer also to the sequence of events within one period.

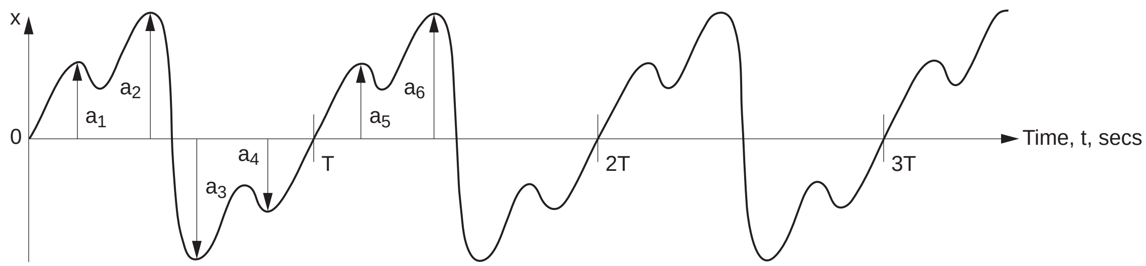

Figure 5 Periodic Motion

AMPLITUDE — Figure 5 shows time history of a vibratory motion, which repeats itself every T seconds. The maximum values of the displacement, x, from the reference position (x = 0) are called PEAKS. These are (a_1, a_2...). The largest of these is called the *PEAK AMPLITUDE.*

STEADY-STATE MOTION — A periodic motion of a mechanical system; e.g., a continuously swinging pendulum of constant amplitude.

STOCHASTIC or RANDOM MOTION — A motion which changes with time in a nonperiodic, possibly very complex, manner.

HARMONICS — Any motion can be considered as made up of a sum (often an infinite number) of simple harmonic motions of different frequencies and amplitudes. The lowest-frequency component is usually called the *FUNDAMENTAL FREQUENCY*; higher frequency components are called *HARMONICS*. Their frequencies are multiples of the fundamental frequency. Sometimes, components with frequencies which are fractions of the fundamental frequency (subharmonics) are significant (e.g., the "half-frequency" whirl of rotating shafts, etc.).

PULSE — Usually a displacement-time or force-time function describing a transient input into a dynamical system.

PULSE SHAPE — The shape of the time-displacement or force-displacement curve of a pulse. Typically, this might be a square wave, a rectangular pulse, or a half sine-wave pulse. In general, however, the shape can be an arbitrary function of the time.

SHOCK MOTION — A motion in which there is a sharp, nearly sudden change in velocity; e.g., a hammer blow on a nail, a package falling to the ground from a height, etc. Its mathematical idealization is that of a motion in which the velocity changes suddenly. This idealization often represents a good approximation to the real dynamic behavior of the system.

2.2 Rigid-Body Characteristics

MASS — Inertia of the body equal to its weight in lbs. or in Newtons (N) divided by the gravitational constant (g = 32.2 ft/sec^2 = 386 in/sec^2 = 9.81 m/sec^2). Unit of mass, if the weight is expressed in N, is a kilogram (kg).

CENTER OF GRAVITY (CENTER OF MASS, C.G.) — Point of support at which a body would be in balance.

MOMENT OF INERTIA — The moment of inertia of a rigid body about a given axis in the body is the sum of the products of the mass of each volume element and the square of its distance from the axis. Units are in-lb-sec^2, or in kg-m^2 for example. Moments of inertia of the standard shapes are tabulated in handbooks. If instead of mass of the element its volume is used, the result is also called a moment of inertia. Depending on the application, mass-, volume-, or area moments of inertia can be used.

PRODUCT OF INERTIA — The product of inertia of a rigid body about two intersecting, perpendicular axes in the body is the sum of the product of the mass (volumes, areas) of constituent elements and the distances of the element from the two perpendicular axes. Units are the same as for the moment of inertia. Tabulations are available in handbooks and textbooks.

PRINCIPAL AXES OF INERTIA — At any point of a rigid body, there is a set of mutually perpendicular (orthogonal) axes intersecting in the C.G. such that the products of inertia about these axes vanish. These axes are called the principal axes of inertia. In a body having axes of symmetry, the principle axes coincide with them. (An axis of symmetry is a line in the body, such that the body can be rotated a fraction of a turn about the line without changing its outline in space).

2.3 Spring and Compliance Characteristics

TENSION — When a body is stretched from its free configuration, its particles are said to be in tension (e.g., a stretched bar). The tensile force per unit area is called the *tensile stress* (Units: lbs/in^2 (psi) or Pascals, 1Pa = 1N/m^2, 1 Mega Pascal (MPa) = 10^6 N/m^2).

COMPRESSION — When a body is compressed from its free configuration (e.g., a column in axial loading), the compressive force unit per area is called the *compressive stress* (Units: lbs/in^2 or Pa).

SHEAR — When a body is subjected to equal and opposite forces, which are not collinear, the forces tend to "shear" the body; e.g., a rubber pad under parallel forces in the planes of its upper and lower faces. The shear force per unit area is called the *shear stress* (Units: lbs/in^2 or Pa). A body can be in a state of tension, compression and shear simultaneously; e.g., a beam in bending.

SPRING CONSTANT — When a helical cylindrical spring is stretched or compressed by x, the displacement x is proportional to the applied force, F (Hook's law). The proportionality constant (k) (Units: lbs/in, N/m) is called the *SPRING CONSTANT* or *STIFFNESS*, F = kx. If the spring deflects in torsion, the units of k are in-lb/rad, lb/deg, N-m/rad. Such springs are called *LINEAR SPRINGS*. More generally, the load and the displacement are not proportional (a *NONLINEAR SPRING*). In such cases stiffness is changing with the changing load and displacement, and k is the ratio of a force increment ΔF to the corresponding displacement increment Δx in the loading process. An important issue for spring materials most often used in vibration isolators, such as elastomeric (rubber) materials, wiremesh materials, etc., is influence of rate of loading on their stiffness. The stiffness constant measured at low rate of loading (frequency of load application < ~0.1 Hz) is called *STATIC STIFFNESS*, k_{st} and the stiffness constant measured at higher frequencies of load application is called *DYNAMIC STIFFNESS*, k_{dyn}. The *DYNAMIC STIFFNESS COEFFICIENT* is defined as $K_{dyn} = k_{dyn} / k_{st}$.

FORCE-DEFLECTION CHARACTERISTIC — This refers to the shape of the force-deflection curve. For the linear spring, it is a straight line through the origin of coordinates (constant k). If, for a nonlinear spring, its stiffness increases with increasing force or displacement (as in many rubber springs loaded in compression), the characteristic is called "hardening nonlinear". If it decreases with force or displacement (e.g., as in a Belleville spring), the characteristic is called "softening nonlinear".

ENERGY STORAGE — This is the area under the force-deflection curve of the spring. It represents the strain energy stored in the spring (Units: lb. in., lb. ft., N•m).

PRELOAD — A spring or other elastic element used in an isolator or in a coupling may or may not be assembled in a condition in which it has its natural, free, or unstretched length. If its assembled length is not its free length, the spring is in tension or compression even before the isolator is loaded by the object weight or the coupling is loaded by the transmitted torque. The amount of this tension or compression is called the preload. When measured in force units, it is a preload force; when measured in deflection from the free position, it is a preload deflection.

ELASTIC (YOUNG'S) MODULUS (E) AND SHEAR MODULUS (G) — These are material properties, which characterize resistance of the material to deformation in tension or in compression (E) and in shear (G). They are defined as the ratio of stress to strain, where strain is the change in length (or deformation) per unit length. E involves tensile or compressive stress/strain and G involves shear stress/strain. Units: lb/in^2, Pa. In many practical applications, especially for metals, E and G are constants within a limit of material stress known as the proportionality limit. Rubber and plastics often do not have a well-defined proportionality limit.

2.4 Damping, Friction and Energy-Dissipation Characteristics

STATIC FRICTION, SLIDING FRICTION, COULOMB FRICTION — These are all terms used for the frictional resistance for sliding of one body relative to another; e.g., a weight dragged along the floor. The frictional force is approximately proportional to the contact force between the two bodies and is opposed to the direction of relative motion. The proportionality constant f is known as the friction coefficient. If a 10 lb. weight is dragged along a horizontal floor with a friction coefficient f = 0.2, the frictional resistance is 0.2 x 10 = 2 lb. Sometimes a distinction is made between the value of the coefficient of friction when motion is just starting after a stationary condition (*STATIC FRICTION*) and its value during motion (*SLIDING* or *DYNAMIC FRICTION*). The coefficient of friction in the latter case is generally lower and changes with the motion velocity, unless it is *DRY* or *COULOMB FRICTION*, wherein the sliding friction coefficient does not depend on velocity. The motion (kinetic) energy is decreasing due to energy dissipation during a sliding process with friction. Thus, frictional connections can be used as dampers.

VISCOUS DAMPING — If, in a damper, the body moves relative to a second body, *VISCOUS DAMPING* refers to a resisting (friction) force which is proportional and opposite to the relative velocity between the two bodies. The proportionality constant is the coefficient of viscous damping, c. Units: force per unit velocity; i.e., lb/(in/sec) or N/(m/sec). Viscous damping is encountered, for example, in hydraulic dashpots and devices which squeeze a liquid through an orifice. The more viscous the fluid, the greater the damping. If c = 0.5 lb/(in/sec) and the body moves at 10 in/sec, the viscous damping force is 0.5 x 10 = 5 lb. Typical example: hydraulic door closers.

MATERIAL or HYSTERETIC DAMPING — such as damping in rubber isolators, wire mesh isolators, etc., depends on vibration amplitudes rather than on vibratory velocity. While both viscous and hysteretic damping reduce resonance amplitudes, the viscous damping spoils vibration isolation efficiency at high frequencies (when vibration amplitudes are decreasing) while the intensity of hysteretic damping automatically decreases with the decreasing amplitudes and it results in a better isolation efficiency.

CRITICAL DAMPING c_{cr} — Value of damping constant in mass-spring-damping system just sufficiently high so as to prevent vibration.

DAMPING RATIO c/c_{cr} — The ratio of the damping constant to the critical damping constant for that system. The damping ratio is related to log decrement δ as

$$\delta = 2\pi\,(c/c_{cr}). \quad (2)$$

2.5 Vibration Characteristics of Mechanical Systems

MATHEMATICAL MODEL — An idealized representation of the real mechanical system, simplified so that it can be analyzed. The representation often consists of rigid masses, springs and dampers (dashpots). The model should be sufficiently realistic so that results of the analysis of the model correspond reasonably closely to the behavior of the physical system from which it was derived.

LUMPED- AND DISTRIBUTED-PARAMETER SYSTEMS — In a lumped-parameter system, the mass, elastic spring and damping properties are separated or lumped into distinct components, each having only mass, only elasticity or only damping, but not more than one of these properties per component. In a distributed-parameter system, a component may possess combined mass, elasticity and damping, distributed continuously through the component. The latter systems represent more realistic models, but are more difficult to analyze.

DEGREES OF FREEDOM — This is the number of independent quantities (dimensions or coordinates), which must be known in order to be able to draw the mechanical system in any one position, if the fixed dimensions of the system are known. The simple mass-spring system of Figure 1 has one degree of freedom; a mechanical differential, for example, has two degrees of freedom; a rigid body moving freely in space has six degrees of freedom (three translational and three angular coordinates should be known in order to fully describe the position of the body in space).

FORCE AND MOTION EXCITATION — If a force varying in time is applied to a dynamical system, it usually is a source of vibration (e.g., centrifugal force due to an unbalanced rotor). The vibrations are then said to be due to force excitation. If, on the other hand, the foundation (or other part) of a machine is subject to a forced motion (vibration or shock), the resulting machine vibration is said to be due to motion excitation; e.g., an earthquake actuating a seismograph.

FREE VIBRATION — If the massive block in Figure 1 is moved out of its equilibrium position, and released, the system will vibrate without the action of any external forces. Such an oscillation is called a free vibration.

FORCED VIBRATION — If an external force is applied to the weight in Figure 1, which causes it to vibrate (e.g., a force varying harmonically with time), the resulting motion of the spring-mass system is called a forced vibration. If the base which supports the spring, undergoes a forced motion which in turn causes the weight to vibrate, the vibration is also forced.

RANDOM VIBRATION — Equipment may be caused to vibrate by applied forces or motions in which frequencies and amplitudes of harmonics vary in a random manner with time (e.g., wind gusts on a missile). The resulting vibration is called random.

NATURAL FREQUENCY — Whether the system is without damping or with damping, the frequency of free vibration is called the free-undamped natural frequency or the free-damped natural frequency. The natural frequency is a function of the mass and stiffness distribution in the system. For a simple-mass spring system, which is a reasonable approximation to many real mechanical systems, the natural frequency, f_n, is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{\frac{g}{x_{st}}} \text{ Hz.} \qquad (3)$$

Here, k is spring constant (dynamic stiffness constant k_{dyn} should be used, see Section 2.3); W is the weight; g is the gravitational constant, 386 in/sec^2 or 9.8 m/sec^2; and x_{st} is the static deflection of the spring. The reciprocal to the natural frequency is the *NATURAL PERIOD* $T = 1/f_n$, sec. If x_{st} is expressed in cm (1 cm = 0.01 m), then the natural frequency can be conveniently found as

$$f_n \approx \frac{5}{\sqrt{x_{st}}} \text{ Hz.} \qquad (4)$$

The angular natural frequency ω_n in radians per second is

$$\omega_n = \sqrt{\frac{kg}{W}} \qquad (5)$$

Thus, flexible systems tend to have low natural frequencies and rigid systems tend to have high natural frequencies. At the same time, the natural frequency can be changed by altering the stiffness and mass distribution of the system. A system may have more than one natural frequency, in which case the lowest of these is often the most significant one. The number of natural frequencies is equal to the number of degrees of freedom of the system. Presense of damping is slightly reducing the natural frequency; The *DAMPED NATURAL FREQUENCY* is

$$f_{dn} = f_n\sqrt{1 - \left(\frac{c}{c_{cr}}\right)^2} = f_n\sqrt{\frac{1-\delta^2}{4\pi^2}} \qquad (3a)$$

where $\delta = 2\pi\,(c/c_{cr})$
c = damping constant
c_{cr} = critical damping constant

FORCING FREQUENCY — The frequency of an external force or motion excitation applied to a vibrating system.

2.5.1 Amplitude-Frequency Characteristics of Forced Vibrations

If a sinusoidal force $F(t) = F_o \sin 2\pi ft$ is acting on massive block W connected with the base by spring having stiffness k and viscous damper with resistance coefficient c, Figure 6, then sinusoidal vibration of block W is excited. If frequency f is changing but amplitude F_o is constant in a broad frequency range, then amplitude of the vibratory displacement of block W changes with frequency along an *AMPLITUDE-FREQUENCY CHARACTERISTIC*, Figure 7. Figure 7 shows plots of the displacement amplitudes vs. *FREQUENCY RATIO* f/f_n for various degrees of damping (*LOG DECREMENT* δ) in the vibratory system. The plots in Figure 7 are described by the following expression for the response amplitude A of the massive block W to the force excitation:

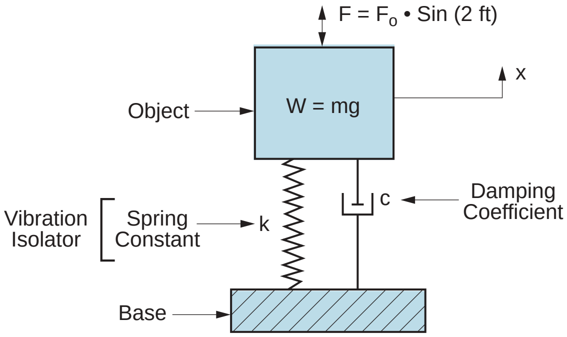

Figure 6 Simple Vibratory System Under Forced Excitation

$$A = \frac{F_o}{k\sqrt{\left(1-\frac{f^2}{f_n^2}\right)^2+\left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}} = \frac{F_o/k}{\sqrt{\left(1-\frac{f^2}{f_n^2}\right)^2+\left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} \quad (6)$$

RESONANCE — It is seen in Figure 7 that displacement and stress levels tend to build up greatly when the forcing frequency coincides with the natural frequency, the build-up being restrained only by damping. This condition is known as *RESONANCE.*

In many cases, the forced vibration is caused by an unbalanced rotating mass, such as the rotor of an electrical motor. The degree of unbalance can be expressed as distance e between the C.G. of the rotor and its axis of rotation. The vertical component of the centrifugal force generated by the unbalanced rotor (mass M) is

$$F_{c.f.} = M\omega^2 e \sin \omega t = 4\pi^2 M f^2 e \sin 2\pi t, \quad (7)$$

where ω is angular speed of rotation in rad/sec and f is the number of revolutions per second. In case of vibration excitation by the unbalanced rotor, combining of (6) and (7) results in

$$A = \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1-\frac{f^2}{f_n^2}\right)^2+\left(2\frac{c}{c_{cr}}\frac{f}{f_n}\right)^2}}$$

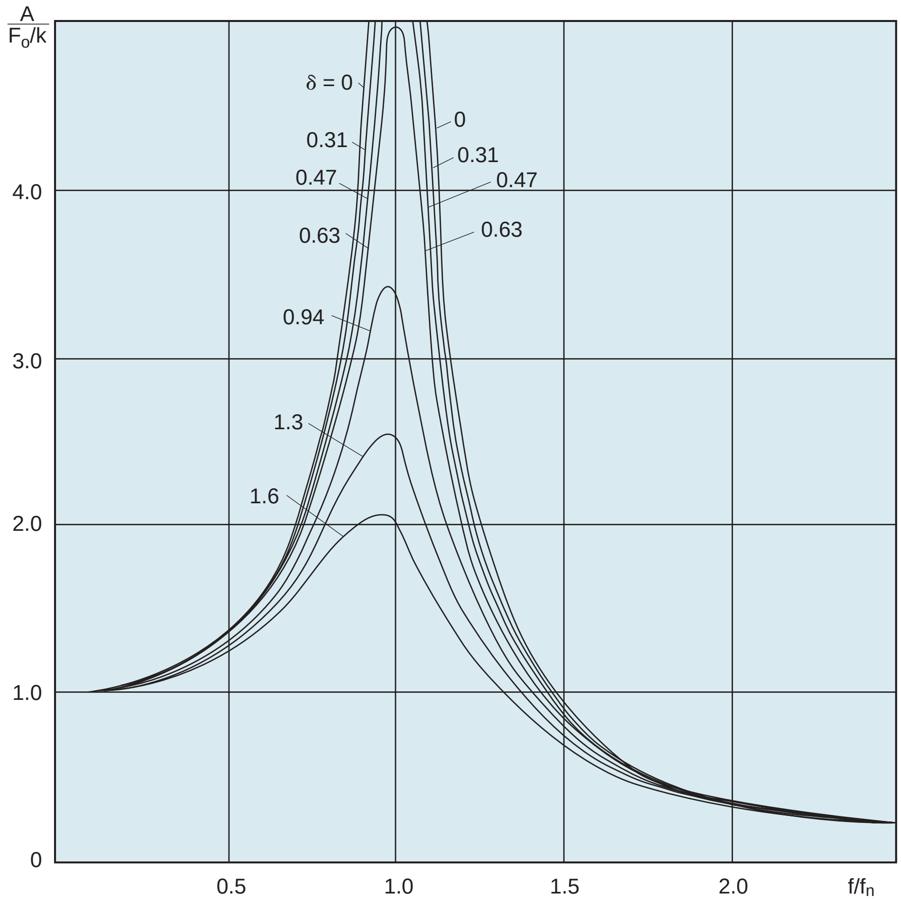

Figure 7 Amplitude-Frequency Characteristics of Massive Block Motion in Figure 6

$$= \frac{4\pi^2 M f^2 e}{k\sqrt{\left(1-\frac{f^2}{f_n^2}\right)^2+\left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}} = \frac{Me}{m}\,\frac{f^2/f_n^2}{\sqrt{\left(1-\frac{f^2}{f_n^2}\right)^2+\left(\frac{\delta}{\pi}\frac{f}{f_n}\right)^2}}, \quad (6a)$$

where m is the total mass of the object. Expression (6a) is plotted in Figure 8 for several values of damping (δ).

3.0 Vibration Isolation

Although *VIBRATION ISOLATION* is a very large area of vibration control, there are two most widely used techniques of vibration isolation:

– Reduction of transmission of vibratory or shock forces from the object, in which these forces are generated, to the base; and

– Reduction of transmission of vibratory motions of the base to the work area of vibration-sensitive objects.

These techniques are similar, but also quite different. They both deal with *TRANSMISSIBILITY* or *TRANSMISSION RATIO*. There are several transmission ratios. Usually these refer to the ratios of the maximum values of the transmitted force or displacement to the maximum values of the applied force or the forced motion. The important direction of transmission is from the object to the base for the force isolation, or from the base to the object for the motion isolation.